



## DYNAMIC MODELING OF TECHNOLOGICAL PROCESSING SYSTEMS

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**Abstract:** The article describes the mathematical modeling results of the dynamics in order to determine the regularity of change in the lathe dynamics during the cutting tool wear. A close correlation between the sound trend accompanying the metalworking process and the roughness trend of the machined surface is shown. The calculation results serve as the basis for solving the problem of operational resource tool prediction. The tool wear operational forecast allows for the first time in the material processing history to put into practice an effective adaptive control technology of the cutting process, which determines the novelty of the material described in the article. Further research should focus on developing a predictive model. The model should be a time function and have a minimum of parameters, which must include the desired cutting tool wear  $T$  as a numerical value.

**Key words:** Processing system, tool wear, sound trend, predictive model, mathematical model, metal-processing technological systems

### Introduction – lathe dynamics during cutting tool wear

The research purpose of the processing system dynamics is to study the regularity of its dynamic behavior. Knowing these regularities allows you to purposefully manage the metalworking process and avoid the appearance of defects in the workpiece.

These regularities appear [1] in:

- the trend and the spectral composition of the sound generated during the materials cutting processing;
- the trend of the roughness height parameter and its profile, which changes during the cutting process due to tool wear.

These parameters have a decisive influence on the metalworking quality. The mathematical description of the lathe elastic system must be connected with the processes occurring in the working area of the processing system [2, 3, 4].

Each adopted dynamic model uniquely corresponds to a certain differential equations system describing its behavior. These equations can be considered as a dynamic system mathematical model. Depending on the type of differential equations, mathematical models can be linear and nonlinear [5].

In a linear dynamic model, the elastic forces are proportional to the deformations, the viscous resistance forces to the velocities, and the inertial forces to the accelerations. The article discusses a similar linear dynamic model of a lathe.



## Modeling Methodology

### The research purpose of the processing systems dynamics

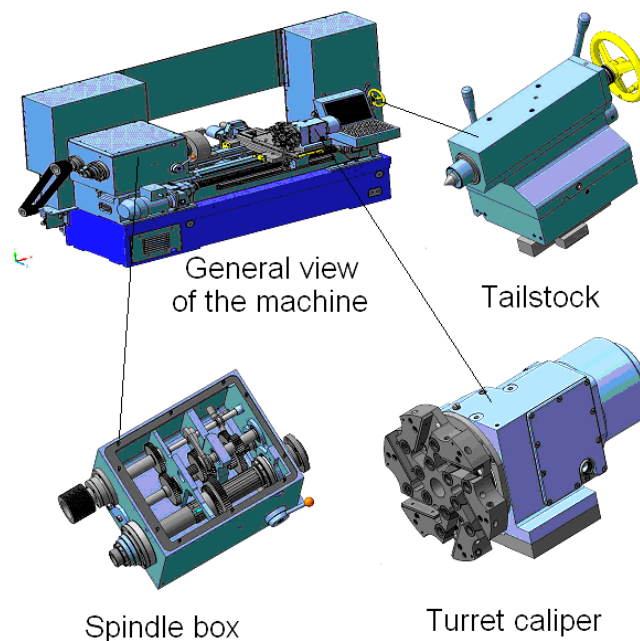
The research purpose was to establish dynamic behavior regularity of the processing system during the technical condition (wear and destruction) of the cutting tool changes and the nature of this regularity in the amplitude sound wave trend accompanying the metalworking machines work.

### Subject of research

The research subject was a typical technological metalworking system dynamic model – a lathe technological system.

### Structural characteristics of the lathe

The lathe model 16K20T1 (Fig. 1) is a machine of high accuracy. It is equipped with a CNC and is designed for processing parts such as rotation bodies according to a program entered manually or recorded on a hard disk.



**Fig. 1 Lathe model 16K20T1**

The lathe base is a rigid casting, on which are installed: a frame, the main movement electric motor, a lubrication station for the guide carriage of the spindle head, a grease-coolant pump. The middle part of the base serves as a collection for shavings and grease-coolant, the compartment in the lower right part of the base serves as a reservoir for grease-coolant.

The lathe frame has a box-like shape with transverse ribs profile, on which hardened polished guides are fixed. The headstock, carriage, longitudinal feed drive, tailstock are installed on the frame. For basing the carriage on the frame the front guide has the form of an unequal prism, the rear guide is flat. The tailstock is based on the frame on the small rear prismatic guide and on the plane on the front guide.



For the processing of billet on the lathe used various types of incisors with mechanical fastening of the cutting plates. Many-sided hard alloy plates are fixed with clamps, screws, wedges, etc. The cutting part materials are also different and depend on the material being processed.

### Research methods

The research technique consisted in:

- computer simulation of the processing system oscillations when changing due to cutting tool wear and destruction, its stiffness and damping characteristics;
- comparing the modeling results and verification experiments to confirm the calculations reliability and to identify regularity of changes manifestation in the dynamics of the processing system in the behavior of the sound wave trend amplitude accompanying its work.

### Lathe Dynamic Model

The mass, stiffness and viscous coefficients of the simulated lathe nodes are indicated on the model diagram, respectively, using  $m$ ,  $k$  and  $c$ . These parameters characterize each simulated machine nodes partial oscillations in the direction perpendicular to the main axis for the simulated node axes of stiffness and damping. The generalized coordinate  $\zeta_i$  describes the spatially oriented oscillations of the mass center of each of the  $i$ -th modeled elements of the technological system.

Differential equations using the complex amplitude method by substitution  $\xi(\tau) = \xi_A \cdot \exp(i\omega\tau)$ , where  $\xi_A$  – is the oscillation amplitude of the mass center of the simulated machine element,  $m$  [184], were transformed into a algebraic equations system with complex coefficients. These algebraic equations system was solved by the Gauss method.

The model diagram is shown in Fig. 2, and its parameters – in Table 1. The oscillations of the model were described by means of six differential equations (1). In this case, the oscillations of the following elements of the technological system were considered: tool blades ( $m_1, k_1, c_1$ ); tool holders ( $m_2, k_2, c_2$ ); machine caliper ( $m_3, k_3, c_3$ ); tool holder (turret) ( $m_6, k_6, c_6$ ); frame with front and back headstock ( $m_4, k_4, c_4$ ); spindle with chuck and workpiece ( $m_5, k_5, c_5$ ).

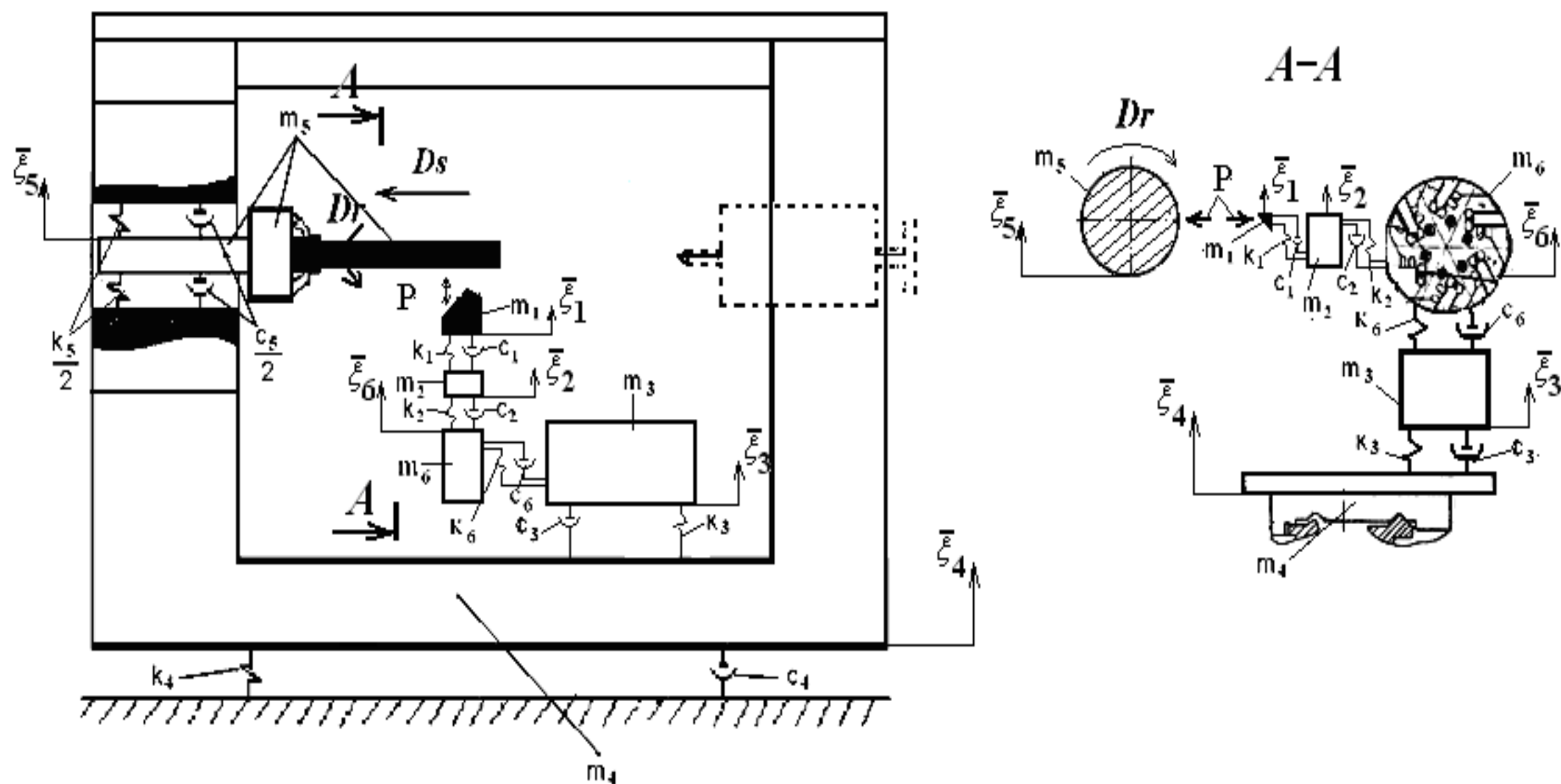


Fig. 2 Dynamic Model of Lathe Process System



**Table 1. Lathe dynamic model parameters**

Dynamic parameter	Simulated technological system node (mass number)					
	blade (1)	tool holder (2)	machine caliper (3)	frame (4)	spindle with chuck and workpiece (5)	cutter-holder (6)
Mass $m_i, \text{kg}$	$0,29 \cdot 10^{-6}$	0,12	37	3 693	29	20
Stiffness coefficient $k_i, \text{N/m}$ ( $f_i, \text{Hz}$ )	$1,96 \cdot 10^8$ ( $4,2 \cdot 10^6$ )	$7,75 \cdot 10^{10}$ ( $1,28 \cdot 10^5$ )	$2,73 \cdot 10^9$ (1 370)	$2,44 \cdot 10^{12}$ (4 083)	$5,47 \cdot 10^{10}$ (6 920)	$3,15 \cdot 10^9$ (2 000)
Viscous resistance coefficient $c, \text{N/m/sec}$ (quality factor Q)	0,5 (20)	2 170 (20)	$4,33 \cdot 10^4$ (10)	$3,84 \cdot 10^6$ (25)	$8,45 \cdot 10^4$ (15)	$6,78 \cdot 10^4$ (15)

Note: Contact force  $P_{CF} = 107 \text{ N}$ ; stiffness k was determined by the following formula:

$k_i = m_i (2\pi f_i)^2$ , where  $f_i$  – is the frequency of natural partial oscillations of the simulated node was calculated by the identification method; damping  $c_i$  was determined by the following formula:  $c_i = \frac{\sqrt{k_i \cdot m_i}}{Q_i}$ , where  $Q_i$  – is the quality factor (value) of the peak of the own partial oscillations of the modeled node was calculated by the identification method.

Differential equations:

$$\begin{aligned}
 &1) m_1 \ddot{\zeta}_1 + c_1(\dot{\zeta}_1 - \dot{\zeta}_2) + k_1(\zeta_1 - \zeta_2) = P(t); \\
 &2) m_2 \ddot{\zeta}_2 - c_1(\dot{\zeta}_1 - \dot{\zeta}_2) - k_1(\zeta_1 - \zeta_2) + c_2(\dot{\zeta}_2 - \dot{\zeta}_6) + k_2(\zeta_2 - \zeta_6) = 0; \\
 &3) m_3 \ddot{\zeta}_3 - c_6(\dot{\zeta}_6 - \dot{\zeta}_3) - k_6(\zeta_6 - \zeta_3) + c_3(\dot{\zeta}_3 - \dot{\zeta}_4) + k_3(\zeta_3 - \zeta_4) = 0; \\
 &4) m_4 \ddot{\zeta}_4 - c_3(\dot{\zeta}_3 - \dot{\zeta}_4) - k_3(\zeta_3 - \zeta_4) + c_5(\dot{\zeta}_4 - \dot{\zeta}_5) + k_5(\zeta_4 - \zeta_5) \\
 &\quad + c_4 \dot{\zeta}_4 + k_4 \zeta_4 = 0; \\
 &5) m_5 \ddot{\zeta}_5 - c_4(\dot{\zeta}_4 - \dot{\zeta}_5) - k_4(\zeta_4 - \zeta_5) = -P(t); \\
 &6) m_6 \ddot{\zeta}_6 - c_6(\dot{\zeta}_3 - \dot{\zeta}_6) - k_6(\zeta_3 - \zeta_6) - c_2(\dot{\zeta}_2 - \dot{\zeta}_6) + k_2(\zeta_2 - \zeta_6) = 0.
 \end{aligned} \tag{1}$$



Algebraic equations:

$$\begin{aligned} 1) & -m_1\omega^2\zeta_1 + i\omega c_1(\zeta_1 - \zeta_2) + k_1(\zeta_1 - \zeta_2) = P(t, \omega) ; \\ 2) & -m_2\omega^2\zeta_2 - i\omega c_1(\zeta_1 - \zeta_2) - k_1(\zeta_1 - \zeta_2) + i\omega c_2(\zeta_2 - \zeta_6) + k_2(\zeta_2 - \zeta_6) = 0; \\ 3) & -m_3\omega^2\zeta_3 - i\omega c_6(\zeta_6 - \zeta_3) - k_6(\zeta_6 - \zeta_3) + i\omega c_3(\zeta_3 - \zeta_4) + k_3(\zeta_3 - \zeta_4) = 0; \\ 4) & -m_4\omega^2\zeta_4 - i\omega c_3(\zeta_3 - \zeta_4) - k_3(\zeta_3 - \zeta_4) + i\omega c_5(\zeta_4 - \zeta_5) + k_5(\zeta_4 - \zeta_5) - \\ & - i\omega c_4\zeta_4 - k_4\zeta_4 = 0; \\ 5) & -m_5\omega^2\zeta_5 - i\omega c_4(\zeta_4 - \zeta_5) - k_4(\zeta_4 - \zeta_5) = -P(t, \omega); \\ 6) & -m_6\omega^2\zeta_6 - i\omega c_6(\zeta_3 - \zeta_6) - k_6(\zeta_3 - \zeta_6) - i\omega c_2(\zeta_2 - \zeta_6) - k_2(\zeta_2 - \zeta_6) = 0. \end{aligned} \quad (2)$$

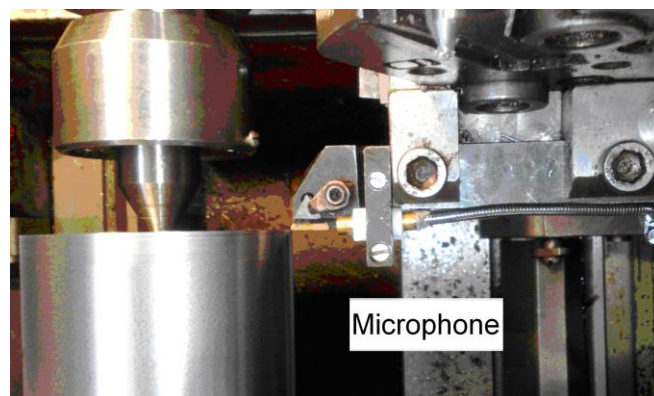
### Research results

The results of experimental computational research are presented in Fig. 4 and 5. They are usually a comparison of calculated and experimental data. The calculation results are obtained when solving the equations system (2). The system solution was carried out repeatedly when changing the cutting time from 0 to  $T$ , equal to the period of durability (resource) tool with a time step  $\Delta\tau = 1 \text{ sec}$ . At the same time, at each time step, the frequency characteristics of the model oscillations were determined with successive changes in frequency from 0 to 2,500–3,000  $\text{Hz}$  with a step in frequency  $\Delta f = 10 \text{ Hz}$ .

To assess the developed model reliability in the calculation in addition to the trend of the sound wave amplitude were investigated:

- the change nature over time of the sound implementation;
- sound trend generated during the cutting process;
- roughness profile;
- roughness profile trend.

The sound was measured using a microphone placed near the cutting zone (Fig. 3).

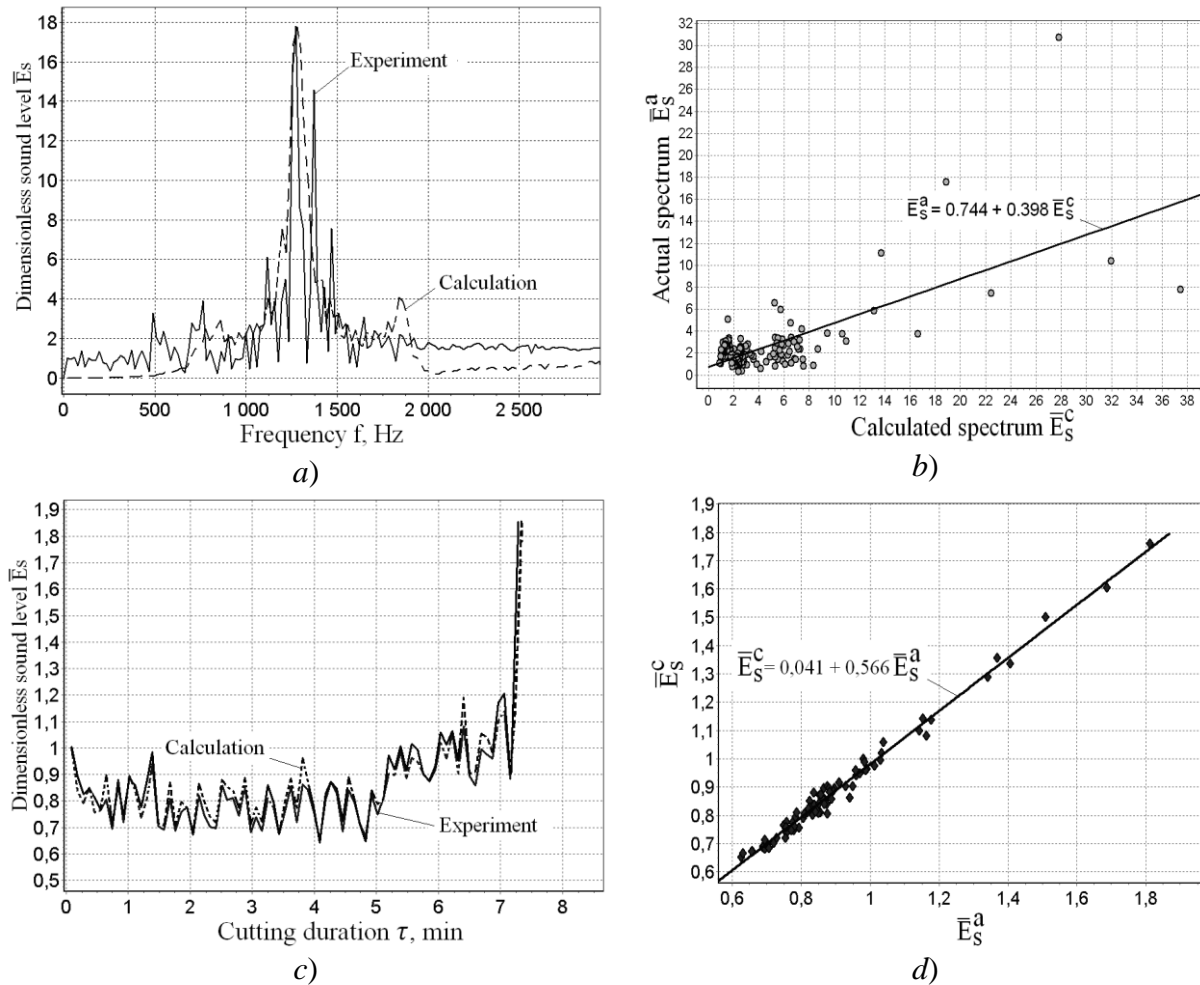


**Fig. 3 Placement the microphone near the cutting area**

Fig. 4a shows the actual and calculated sound spectra accompanying the cutting process, which visually coincide quite well with each other. This is also indicated by the quantitative degree assessment of their coincidence, described by their correlation coefficient, equal to 0.684 (Fig. 4b). The most important are the calculations describing the nature of the trend in sound



$\bar{E}_S$  (Fig. 4c) as the cutting blade wears. Correlation coefficient between calculated and experimental sound trend values  $\bar{E}_S$  equals  $R = 0,993$  (Fig. 4d). This information shows that the model reproduces well enough not only the frequency sound filling, but also describes the trend change nature of the sound over time as the instrument wears.

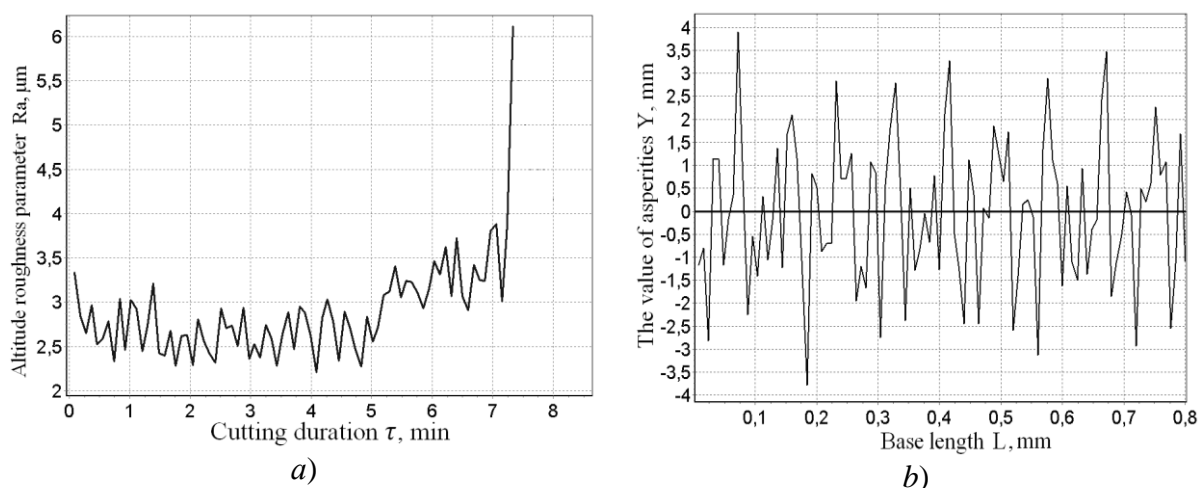


**Fig. 4 Results of calculating the spectrum and trend of sound accompanying cutting processing: a - comparison the calculated and actual sound spectra; b - regression line between the actual and calculated spectra; c) comparing actual and calculated sound trends; d) regression line between actual and estimated sound trends**

Fig. 5 shows the calculated trend of the altitude parameter  $Ra$  roughness and the roughness profile calculation.

At the same time, a fundamentally important result of calculations is the determination of the fact that sound trends  $\bar{E}_S$  (Fig. 4c) and roughness (Fig. 4a) are identical.





**Fig. 5 Roughness parameters obtained by the calculated method:**  
*a* – is trend of altitude parameter roughness *Ra*; *b* – is roughness profile *Y*

## Conclusion

The mathematical modeling results of the lathe dynamics serve as the basis for solving the problem of operational tool life prediction. This problem solution allows for the first time in the processing materials history to put into practice an effective technology of adaptive cutting process control.

To implement this technology, tool life prediction should be performed in real time directly during the processing of materials by cutting. In this case, the prediction technique should be based on the predictive model, which should be a time function and have a minimum of parameters, which must include the desired durability of the cutting tool *T* as a numerical value.

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