USING OF BRANCH AND BOUND TO MANAGING OF STOCKS IN DEPEDENCE ON MECHANICAL PRODUCTION DEMAND

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Abstract

The main idea of article is the calculation of optimal amount of stocks. It assumes the consecutive solving in deterministic model with unbalanced demand. Optimalization is performed by methods of branching and bounds. The question arises how many orders are required for the future consumption and how big should be each order to achieve minimal costs for the sum of cost of order and carrying-costs of storage cost.

Key words: algorithm, production, management, components, stocks, demand

1. INTRODUCTION

The target of this article is to define an optimal strategy for purchasing of material. That means to define how many quantitative units will cover needs in certain time. We used in the case study example absolutely determined by unequal consumption (the method of branching and bounds).

2. THE MODEL OF MANAGING OF SUPPLIES

The elementary task in theory of stock is to determine the optimal amount of stock to ensure continuous course of production with minimal costs. The main division of stock models:

- deterministic models
- stochastic models

By deterministic models are known moments of supplying, consumption, time of delivery and so on. Controllability of the system means that system which controls the stock can determine according to the need the scale and time of division of each value. [2]

By stochastic models one of the values is random for example delivery shift, consumption or time. On the basis of strategy of filling up the stocks i.e. rules according to which we determine the amount of delivery and time of delivery are divided to:

- models with signalizing the changes,
- models with periodical control
- The elementary model of theory of stock consists of three parts:
- mechanism of entry
- mechanism of storage
- mechanism of output

The mechanism of entry to system represents the group of rules according to which we fill the stock with deliveries.

We differ from four types of delivery:

- deterministic delivery assigned to size and time
- deterministic delivery assigned to size but randomly divided in time
- delivery randomly assigned to size but deterministic divided in time
- delivery randomly assigned to size and time

The mechanism of storage represents the group of rules which assign in which way the inputs are changed to outputs. These are decision rules which control operation of storage system. The prevailing numbers of storage models presume that the aim is to achieve the maximum level of service provided minimal costs. So it is the costs which make the point.

The mechanism of output represents the group of rules according to which the stored components are used up within the company or as a delivery from outside of the company.

2.1 Deterministic models of stock

These models are based on the principle of total informedness and total controllability. Total informedness means we know all the basic elements of the model such as moment of delivery, consumption, time of delivery and so on. The total controllability assumes that system which controls the store can determine the size and time division of individual values according to the need.

To say in simple words, we suppose that we need to fill up the stores of material that is used continuously e.g. propellant. The storage has certain costs as well as delivery, but the loss arises as a result of the lack of material (non operating state in production, losing of customers). The question is when and how much of stock we should fill up to achieve the minimal costs on storage, filling up and the lack of material.

We can imagine a simple model in which we assume:

- there is no restriction to dividing of material
- consumption is uniform and well- known, λ units in one unit of time
- the amount of delivery (Q) in the store is not limited, we can choose the moment of delivery in any time
- the time of storage is not limited and material is not getting too old
- the costs of one delivery is not depended on its amount
- it is not allowed to admit the lack of material on the store during the in trivial time period. [6]

2.2 Stochastic models of supplies

Only few real storage systems act deterministic. At least some of these factors – consumption, time of delivery, amount of delivery are usually random – stochastic. Stochastic models are divided according to more aspects. The aspect of filling up of stock is the most important i.e. rules on the base of which we determine the amount of delivery and time of delivery or time of setting of order. [2]

2.3 Stochastic model with signalization of change

More of the variables that occur in the theory of stock such as consumption, time of delivery can be random. Because of that we can not plan filling up the stocks at the time when we run out of them. That is why we solve the filling up by 2 ways: [6]

1. Delivery is ordered when the stocks descent at the given amount (system with signalizing)
2. The amount of delivery is controlled automatically (system with the periodical control)

In the following text we will take into account random consumption, whereas time of delivery will be considered deterministic.

Used substitution:

λ - (random) consumption per unit of time,

cd; cs – costs of delivery or storage of unit of goods in the unit of time

r – the level of stocks by which we realize delivery

Q – delivered amount

ρ – average level of rest of stocks in time of delivery

δ – length of term of delivery

Supposed that the stocks are running out equally that means average consumption per period t is t λ;

where $\overline{\lambda} = E(\lambda)$.

Then average level of stock is $\rho + \frac{1}{2}Q$ a

$\rho = r - \delta\overline{\lambda}$

Supposed ρ > 0; i.e. r > δ λ; then the total average costs are:

$$C(r,Q) = c_s\left(\frac{1}{2}Q + r - \delta\overline{\lambda}\right) + c_d\frac{\overline{\lambda}}{Q} + D(r,Q) \quad (1.3),$$

where D(r,Q) is the loss caused by random shortage.

In the case of random consumption there are also other interesting tasks, e.g. how should be the stock, to satisfy demand with 90% probability. [6]

2.4 Stochastic model with periodical control

The role of stochastic theory of stock is with signalizing is constant monitoring of the amount of stock. Another disadvantage is that it is not possible to coordinate delivery of more goods. To solve these problems it is necessary to fill up the stock in regular time intervals according to stock which is in the store. We will control the amount of stock in the equivalent moments that are distant T time units from each other and we will fill them up them up at the level R. We will keep our assumption of consumption and time of delivery from the paragraph 2.3.1. . If the first delivery is in the time 0, in time δ will be the amount of stock equal to R - λ_δ and T +in time δ before delivery of time R – $\lambda_{T+\delta}$. The average amount of stock during one cycle will be

$$\frac{R - \overline{\lambda}\delta + R - \overline{\lambda}(T+\delta)}{2} = R - (\delta + T/2)\overline{\lambda}$$

The total expected costs per unit of time will be

$$C(R,T) = c_s\left(R - \overline{\lambda}(\delta + T/2)\right) + c_d\frac{1}{T} + D(R,T)$$

where D(R, T) is the loss caused by shortage.

3. THE METHOD OF BRANCHING AND BOUNDS

- **The principle of branching –** the set of acceptable solutions (MPR) is decomposed to series of disjunctive subsets.
- **The principle of estimation of boundaries –** it refers to estimation of criterial function on MPR i.e. on any of its subsets i.e. upper and lower boundary [5]
- **MPR** is the short cut of the set of acceptable solutions.

The procedure (for the maximizing of criterial function): [5]

1. We consider the total MPR the only candidate of branching and we assess the lower boundary of

criterial function fs = -∞ (it has been the best solution, globally valid) and the upper boundary of criterial function on MPR or on any of its subsets f_H (it is valid only for the given (sub)set of solutions) its estimation will be dependent on the character of the assignment (it is the biggest theoretically possible value of criterial function for the solution from MPR.

2. Branching – we will choose one set from the candidates according to the chosen rule of branching (e.g. the highest value f_H) and we will decompose it to one or more subsets of acceptable solutions.

- If the group of candidates is empty, the algorithm will finish. The solution referring to actual lower boundary fS is optimal i.e. $f_S = f(\bar{x}) \Rightarrow \bar{x}$ is optimal solution.
- If $f_S = -\infty$ then solution does not exist (MPR is empty).

3. Assessing of the upper boundary f_H - we assess for each new subset the upper boundary of criterial function on this subset (it is the best theoretically possible value of criterial function for the solution from given subset).

4. Trimming – from further examination we exclude those subsets for which fH < fS or which are empty.

5. Assessing of the lower boundary f_S – we assess the best acceptable solution $\bar{x}$ for each new subset:

- If $f(\bar{x}) \geq f_S \Rightarrow$ we adjust actual lower boundary $f_S = f(\bar{x}), \bar{x}$ is the best found solution and we will afresh perform the trimming whereby we will find out if it is not possible to exclude further subsets on the base of condition from step 4.

6. Return to step 2.

3.1 Absolutely deterministic, unequal consumption

We face the situations by assembly of difficult equipments (in building industries and so on) when future consumption of the specific item of stock during the whole planned period T is absolutely well known but it is unequal. We suppose that we fill up the stock of this item with the orders from the suppliers (or from own production) and there are certain costs connected with every order (we will mark them the cost of order) independent on size of order. However it arises by storage of each unit of amount given item of stock the carrying-costs of storage costs it means they are directly proportional the time of storage. The question arises how many orders are required for the future consumption and how big should be each order to achieve minimal costs for the sum of cost of order and carrying-costs of storage cost. We will find the answer with the help of dynamically programming. [11]

We will mark the costs for one of order Cs and the carrying-costs unit of amount given material in the storage per unit of time C1. We suppose that amount Qi of given item of stock which is to be consumed in i-type of period must be in the storage by the beginning of this period. It is possible to order this amount to be available in the storage one, two or more periods before the beginning of the period of the i-type. It means that to find an optimal strategy of filling up the stocks it is not necessary to take into account carrying- costs arising from storage from the beginning of this i-type period to the moment of the real consumption of the amount consumed in i type period. These costs arise independently on the size of individual orders. It practically means that the carrying costs will be counted only from the stocks kept in the storage at least one total consumption period. It means that the size of order in i type period will come into following values [11]:

$x_i = Q_i$, $x_i = Q_i + Q_{i+1}$,......., $x_i = Q_i + ...+ Q_n$ i = 1, 2,....., n.

Other size of order is disadvantageous. If the size of order which is to cover the consumption in several periods was assessed not to cover consumption completely in the last period. It would be necessary to make the new order by filling up the stock to the required level before beginning of the last period. In this case it would be more advantageous to provide the whole consumption of the given period by one order. We would achieve decreasing of the average stock coming from the previous order at the same level of order costs.[11]

It is necessary to have the stock in the storage at last at level of the assumed consumption Qi in this period in the beginning of i type period expressed by the time t. The time is measured in of time i=1, 2,...., n. N(i,j) expresses the sum of order costs which is to satisfy consumption in periods i...j and carrying – costs having arisen by this order. Here we have the equation: [11]

$$n(i,j) = C_s + C_1 \sum_{k=i}^{j} (t_k - t_i) Q_i , \quad i = 1, 2, .. n, \; j = i, i+1, .., n$$

Our aim is to assess the size of individual orders to achieve the minimal costs including order costs, carrying costs for the given period t. If F(j) are minimal costs for ensuring necessary stock to provide assumed consumption from the beginning of the planned period until period of j type including, then we can write [11]

$$f(j) = \min_{1 \leq i \leq j} [n(i,j) + f(i-1)], \qquad (1.6),$$

$$\text{whereas} \quad f(1) = Cs. \qquad (1.7),$$

From the recurrent relations (1.6) a (1.7) can be calculated f(2), f(3), ... , f(n), The last item of this sequence expresses minimal obtainable stock costs for the whole planned period.

4. CASE STUDY

Assumed consumption of the certain item of stock for the following year is:

Table 1: Table of consumption of stock

	Period (i) 1 2 3 4 5 6 7 8	Total
The beginning of i-type period (measured in months from the beginning of the planned year)	0 3 4 6 7 8 11 12	
The size of consumption, Q_i	20 30 60 20 50 70 40 10	300

The costs connected with one order (independent on its size) are 150 money units. The storage costs per unit of amount in one month are in the considered case 2 money units. We are to assess optimal number of orders and the size of individual orders, if the interval of acquiring of stocks is 6 weeks.

We must calculate the table of values n(i,j) in order to use the recurrent relations (1.6) and (1.7).

Table 2: Table of values n (i, j)

j \ i	1	2	3	4	5	6	7	8
1	150							
2	330	150						
3		270	150					
4		390	230	150				
5		790	530	250	150			
6				530	290	150		
7					610	390	150	
8							170	150

The values in this table are calculated according to following equation: (1.5):

$$n(i,j) = C_s + C_1 \sum_{k=i}^{j} (t_k - t_i) Q_i$$

n(1,1) = 150

n(1,2) = 150 + 2(3 – 0) * 30 = 330

n(2,2) = 150

n(2,3) = 150 + 2(4 – 3) * 60 = 270

n(2,4) = 150 + 2{(4 – 3) * 60 + (6 - 3) * 20} = 390

……,

……,

……,

n(6,7) = 150 + 2(11 - 8) * 40 = 390

n(7,7) = 150

n(7,8) = 150 + 2(12 - 11) * 10 = 170

Data in the table express costs n(i, j). E.g. n(2,3) = 270 which lies in the intersection point of the line j=3 and column i=2 means that if we order together the amount necessary for the covering of consumption in the 2-nd and 3 rd month, the order costs for the whole amount will arise and the carrying- costs for stocks for the period of 1 month for the covering of at the 3 rd month at the level of 270 money units. [11]

The calculation is going on in every column as long until the difference btw. two consecutive columns does not exceed 150. The difference higher than 150 in two consecutive items of the same column means that carrying costs for stock for the covering of consumption are higher than 150. So it is more advantageous to cover this consumption by a new order, which will come into storage in the beginning of the consumption period. So arises the maximal time horizon which can be covered by the only order. In the table it is marked by outlining of the relevant item. E.g. the data „390“ in the second column means that the order which would come into storage in the beginning of the second period and would big enough to cover also the consumption of the given item in the second and the third period will create the total costs in the level of 390. If we wanted to cover also the consumption of the following period by this order, the related increase of order by 50 units of amount would lead to increase of costs by 400 money units. [11]

The calculation of values f(j) pre j = 1, 2, ..., 8:

$$f(j) = \min_{1 \le i \le j} [n(i,j) + f(i-1)]$$

f(1) = 150

$$f(2) = \min \begin{Bmatrix} n(1,2) \\ n(2,2) + f(1) \end{Bmatrix} = \min \begin{Bmatrix} 330 \\ 150+150 \end{Bmatrix} = 300$$

….,

….,

$$f(8) = \min \begin{Bmatrix} n(7,8) + f(6) \\ n(8,8) + f(7) \end{Bmatrix} = \begin{Bmatrix} 170 + 820 \\ 150 + 970 \end{Bmatrix} = 990$$

Minimal obtainable costs are f(8) = 990. Optimal strategy of obtaining of stocks leads to orders:

1. 20 units of amount to cover consumption in 1. period,
2. 30 units of amount to cover consumption in 1. period,
3. 80 units of amount to cover consumption in 3. and 4. period,
4. 120 units of amount to cover consumption in 5. and 6. period,
5. 50 units of amount to cover consumption in 7. and 8. period.

As the interval of obtaining of stocks is 6 weeks and also the total planning period begins with January, it is necessary to:

1. The 15th of November to send the order of 20 units of amount,

2. The 15th of January to send the order of 30 of amount,
3. The 15th of February to send the order of 80 units of amount,
4. The 15th of May to send the order of 120 units of amount,
5. The 15th of September to send the order of 50 units of amount.

5. CONCLUSION

The method of solving of stocks in mechanical production is a complex problematic which we can not take blithely. It is important to take into account the size of the company, number of series in production and so on. We have to divide our approach to managing the stocks according to deterministic or stochastic models. The problem seems to be simple by deterministic approach while the delivery terms are short, offer of materials and stocks are on the market in required quality and suppliers are reliable because sometimes we need a few days for delivery. Logistic problem will arise if one of the conditions fail or it is profitable to accumulate the orders due to bulk discounts, transportation and so. We can use deterministic model in both cases with unequal demand. The inputs are assigned from previous analyses and optimalization we carry out by method branch and bound what we illustrated in article.

The problem of supplying is difficult in production system with service status for order production where the requirements for inputs have stochastic status and possible use in smaller production systems.

This project will be programmed into integrated software package, where the whole program will be synergically built on three autonomous programs. The first program will ensure expert calculations of technological times for machining based on method of primitives. The second program will ensure managing of stocks from production point of view. It will use deterministic approach with unequal demand where the method branch and bound will be used for assignment of optimal number of the orders and size of particular orders.

The optimalization according to the suppliers will be made in the second step also by method branch and bound. The third program will ensure algorithm which on the basis of initialization formula calculates the shortest time for machining production batch or whole order. This software package will work with the help of interface such as autonomous universal system on existing systems and will be tested in real mechanical production.

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