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APPLICATION OF NEW MATHEMATICAL AND STATISTICAL METHODS FOR SETTING OUT NETWORKS OF ROAD BRIDGE OBJECTS

PRIMENA NOVIH MATEMATIČKIH I STATISTIČKIH METODA ZA POSTAVLJANJE MREŽA DRUMSKIH MOSTNIH OBJEKATA

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Abstract: *The present contribution demonstrates benefit of fusion least square method with other modern statistical methods at a practical example of setting out network of highway bridge. Robust estimation of 1st order parameters realized for example by Danish method provides the ability to better detect possible blunders in conjunction with the 2nd order parameter estimation computed by MINQUE method.*

Key words: *bridge, setting out network, Method of least squares , MINQUE, Danish method, blunders*

Apstrakt: *Ovaj rad pokazuje korist od spajanja metode najmanjih kvadrata sa drugim savremenim statističkim metodama na praktičnom primeru postavljanja mreže mosta na autoputu. Gruba procena parametara prvog reda izvršena putem, na primer, danske metode, omogućava da se lakše otkriju moguće greške u vezi sa procenom parametara drugog reda izračunatih putem metode MINQUE (objektivna ocena putem uslova minimuma).*

Ključne reči: *most, postavljanje mreže, Metoda najmanjih kvadrata, MINQUE, Danska metoda, grube greške*

1 INTRODUCTION

A social concern of each State is to build and maintain a quality transport infrastructure. Modern, efficient transport and safe road network is the primary prerequisite for regional development, trade, investment in productive activities and creating jobs.

Development of the Slovak economy gives rise to the intensity of car traffic on state roads and creates legitimate concerns for the speedy completion of the network of motorways and expressways. The transport network, given by mountainous nature of Slovakia, has overcome many natural obstacles using tunnels or bridges.

1 UVOD

Glavna društvena briga svake države jeste da izgradi i održava kvalitetnu saobraćajnu infrastrukturu. Savremen, efikasan transport i mreža bezbednih puteva je primarni preduslov za regionalni razvoj, trgovinu, ulaganje u proizvodne delatnosti i otvaranje radnih mesta.

Razvoj slovačke privrede dovodi do pojačanog intenziteta automobilskog saobraćaja na državnim putevima i stvara opravdanu zabrinutost za brz završetak izgradnje mreže magistralnih puteva i autoputeva. Transportna mreža, imajući u vidu brdovit teren Slovačke, savladala je mnoge prirodne prepreke pomoću tunela ili mostova.

2 THE SETTING OUT NETWORK OF ROAD BRIDGE OBJECT

A bridge replaces solid body of the road in place where it is necessary to overcome natural or man-made obstacle. High quality building materials and development of advanced methods of construction of bridges give designers the ability to design these objects more beautiful, higher and longer. Those facts are inevitably reflected in the increasing requirements on the range and quality of geodetic works during their building.

It is clear that the required accuracy of staking-out of characteristic points of the bridge, and of the detailed points that define the shape and size of the bridge is possible only by using quality setting-out network (Ižvoltová, 2010).

Setting-out network of road bridge object is usually designed as a local geodetic network in its own coordinate system. Measured distances are not reduced to the national cartography projection, but they are reduced to mean value of top of abutments (Weiss at all, 2001).After this manner derived coordinates ensure required dimensional consistency between project and its embodiment in reality. From the viewpoint of maintaining high quality geodetic work is equally important to pay attention to the physical implementation of setting out network points.
Good results can be achieved when using an in-depth stabilization, which eliminates errors from centration surveying instruments and greatly suppresses the effects of changing atmospheric conditions during the measurement.

In Slovakia, it is recommended that stabilization extends to a depth of at least 4.5 m below ground level.

3 ADJUSTMENT OF MEASUREMENTS IN SETTING OUT NETWORK OF BRIDGE OBJECTS AND ESTIMATE OF ACCURACY CHARACTERISTICS OF THE DEVICES

Solving geodetic task, which aims is to determine the coordinates of setting out network points and to establish their accuracy from adjustment from redundant measurements will be presented on example of bridge object 207-00 "Jánošíkova Studnička" in the section Hybe Važec of highway D1 in Slovakia.

2 USPOSTAVLJANJE MREŽE OBJEKTA DRUMSKOG MOSTA

Most zamenjuje čvrsto telo puta na mestu gde je potrebno savladati prirodnu prepreku ili prepreku izgrađenu ljudskom rukom. Visoko kvalitetni građevinski materijali i razvoj naprednih metoda izgradnje pružaju projektantima mogućnost da projektuju lepše, više i duže objekte. Te činjenice se neizbežno ogledaju u sve većim zahtevima u pogledu obima i kvaliteta geodetskih radova tokom njihove izgradnje.

Jasno je da je tražena tačnost obeležavanja karakterističnih tačaka mosta, i pojedinačnih tačaka koje određuju oblik i veličinu mosta moguća samo pomoću kvalitetnog uspostavljanja mreže (Ižvoltová, 2010).

Uspostavljanje mreže objekta - drumskog mosta se obično projektuje kao lokalna geodetska mreža u sopstvenom koordinatnom sistemu. Merene razdaljine se ne svode na nacionalnu kartografsku projekciju, već se svode na srednju vrednost vrhova krajnjih tačaka (Weiss at all, 2001).

Nakon ovakvog svođenja, koordinate omogućavaju traženu dimenzionalnu konzistenciju između projekta i njegovog realizovanja. Sa stanovišta održavanja visokokvalitetnog geodetskog rada, jednako je važno obratiti pažnju na fizičku implementaciju postavljanja mrežnih tačaka.

Dobri rezultati mogu se postići kada se koristi dubinska stabilizacija, koja eliminiše greške instrumenata za nadzor centrisanja i da u velikoj meri eliminiše delovanje efekte promenljivih atmosferskih uslova tokom merenja.

U Slovačkoj se preporučuje da stabilizacija dostiže dubinu od najmanje 4,5 m ispod površine zemlje.

3 IZRAVNANJE MERNIH VELIČINA PRI USPOSTAVLJANJU MREŽE OBJEKATA MOSTOVA I OCENA KARAKTERISTIKA UREĐAJA U POGLEDU TAČNOSTI

Na primeru objekta - mosta 207-00 "Jánošíkova Studnička", na deonici Hybe Važec autoputa D1 u Slovačkoj, biće prikazano rešavanje geodetskog zadatka, koji ima za cilj da se odrede koordinate uspostavljanja mrežnih tačaka i da se utvrdi njihova tačnost od izravnanja redundantnih merenja.

Mathematical and statistical basis for *n* measurements and *k* estimated coordinates is well-known regular Gauss-Markoff model of full rank of the form (Ghilani, 2006, Caspary, 1987):

$$\begin{aligned} \boldsymbol{v} &= \boldsymbol{A}\boldsymbol{d}\hat{\boldsymbol{C}} - \boldsymbol{dl} \\ \boldsymbol{\Sigma}_l &= \sigma_0^2 \boldsymbol{Q}_l \end{aligned} \tag{1}$$

Symbol $\boldsymbol{Q}_l$ denotes the cofactor matrix of observations $\boldsymbol{l}$, σ_0^2 apriori variance factor and $\boldsymbol{dl}=\boldsymbol{l}-\boldsymbol{l}_0$ vector of reduced observations $\boldsymbol{l}_0=\boldsymbol{f}(\boldsymbol{C}^0)$. n×k - dimensional matrix of known coefficients, referred to as $\boldsymbol{A}$ (Sokol, 2002; Kalatovičová, Gašincová, 2010),

$$\boldsymbol{A} = \left\{ \frac{\partial f(\boldsymbol{C})}{\partial \boldsymbol{C}} \right\}_{\boldsymbol{C}=\boldsymbol{C}^0}.$$

Solution of overdetermined model (1), *n>k*, standard for condition

$$\boldsymbol{v}^T \boldsymbol{Q}_l^{-1} \boldsymbol{v} = min. \tag{2}$$

is for $\forall j \in (1,k)$ linear unbiased

$$E(\hat{C}_j) = C_j \tag{3}$$

and associated effective

$$s_{\hat{c}_j^2} = min. \tag{4}$$

parameter estimate vector (Caspary, 1987; Sokol, 2008)

$$\hat{\boldsymbol{C}} = \boldsymbol{C}^0 + \boldsymbol{d}\hat{\boldsymbol{C}}, \tag{5}$$

$$\boldsymbol{d}\hat{\boldsymbol{C}} = (\boldsymbol{A}^T \boldsymbol{Q}_l^{-1} \boldsymbol{A})^{-1} \boldsymbol{A}^T \boldsymbol{Q}_l^{-1} \boldsymbol{dl}. \tag{6}$$

According to the criteria (3) a (4) is best invariant quadratic unbiased estimationr of the variance σ_0^2 a posterior variance factor

$$s_0^2 = \frac{\boldsymbol{v}^T \boldsymbol{Q}_l^{-1} \boldsymbol{v}}{f} \tag{7}$$

Denominator of relationship (7) *f=n-k* indicates the number of redundant measurements, (Sabová, Gašincová, 2002) i.e. redundancy of model (1).

Matematička i statistička osnova za n merenja i k ocenjenih koordinata je poznat pravilan Gaus-Markovljev model u potpunom obliku (Ghilani, 2006, Caspary, 1987):

$$\begin{aligned} \boldsymbol{v} &= \boldsymbol{A}\boldsymbol{d}\hat{\boldsymbol{C}} - \boldsymbol{dl} \\ \boldsymbol{\Sigma}_l &= \sigma_0^2 \boldsymbol{Q}_l \end{aligned} \tag{1}$$

Simbol $\boldsymbol{Q}_l$ označava kofaktor matricu opažanja $\boldsymbol{l}$, σ_0^2 apriori varijans faktor i $\boldsymbol{dl}=\boldsymbol{l}-\boldsymbol{l}_0$ vektor redukovanih opažanja $\boldsymbol{l}_0=\boldsymbol{f}(\boldsymbol{C}^0)$. n×k - dimenzionalnu matricu poznatih koeficijenata, koju smo nazvali $\boldsymbol{A}$ (Sokol, 2002; Kalatovičová, Gašincová, 2010),

$$\boldsymbol{A} = \left\{ \frac{\partial f(\boldsymbol{C})}{\partial \boldsymbol{C}} \right\}_{\boldsymbol{C}=\boldsymbol{C}^0}.$$

Rešavanje prethodno određenog modela (1), n>k, standardnog za uslov

$$\boldsymbol{v}^T \boldsymbol{Q}_l^{-1} \boldsymbol{v} = min. \tag{2}$$

je za $\forall j \in (1,k)$ lineraran nepristrasan

$$E(\hat{C}_j) = C_j \tag{3}$$

i povezan efektivan

$$s_{\hat{c}_j^2} = min. \tag{4}$$

Vektor ocene parametara (Caspary, 1987; Sokol, 2008)

$$\hat{\boldsymbol{C}} = \boldsymbol{C}^0 + \boldsymbol{d}\hat{\boldsymbol{C}}, \tag{5}$$

$$\boldsymbol{d}\hat{\boldsymbol{C}} = (\boldsymbol{A}^T \boldsymbol{Q}_l^{-1} \boldsymbol{A})^{-1} \boldsymbol{A}^T \boldsymbol{Q}_l^{-1} \boldsymbol{dl}. \tag{6}$$

Prema kriterijumima (3) i (4) najbolja nepristrasna procena nepromenljivih kvadrata varijanse σ_0^2 aposteriori varijans faktora

$$s_0^2 = \frac{\boldsymbol{v}^T \boldsymbol{Q}_l^{-1} \boldsymbol{v}}{f} \tag{7}$$

Imenilac relacije (7) *f=n-k* pokazuje broj redundantnih merenja, (Sabová, Gašincová, 2002) tj. redundantnost modela (1).

The accuracy of the estimated coordinates of the network, respectively construction of various numerical indicators of accuracy and *k* - confidence interval of large areas is determined from the covariance matrix of geodetic tasks (Šíma, 2008; Sabová, Gašincová, 2002; Lipták, 2009)

$$\mathbf{\Sigma}_{\hat{\Theta}} = s_0^2(\mathbf{A}^T\mathbf{Q}_1^{-1}\mathbf{A})^{-1} = s_0^2\mathbf{Q}_{\hat{\Theta}}\,, \qquad (8)$$

where $\boldsymbol{Q}_{\Theta}$ is $k\times k$ cofactor matrix of unknown parameters. In terms of interpretation of the results of adjustments, it can be placed on the most important outputs of the estimation procedure:

n×n cofactor matrix of residuals

$$\boldsymbol{Q_v} = \boldsymbol{Q_l} - \boldsymbol{A}(\boldsymbol{A}^T\boldsymbol{Q}_l^{-1}\boldsymbol{A})^{-1}\boldsymbol{A}^T = \boldsymbol{Q_l} - \boldsymbol{A}\boldsymbol{N}^{-1}\boldsymbol{A}^T, \quad (9)$$

$n\times n$ redundancy matrix of observables $\boldsymbol{l}$

$$\boldsymbol{R} = (\boldsymbol{Q}_l - \boldsymbol{A}\boldsymbol{N}^{-1}\boldsymbol{A}^T)\boldsymbol{Q}_l^{-1}, \qquad (10)$$

Set of measured data $\boldsymbol{l}$ in a horizontal geodetic networks is combination of terrestrial angular and distance measurements made with total stations, which can be usefully combined with vectors measured by GNSS (GPS) technology (Gašincová, 2007; Kalatovičová, 2010). It is very important to know the characteristics of precision geodetic data files as well as possible for computation of confidence intervals of unknown parameters, correct and unbiased statistical interpretation of measurements results. Estimation of variance components of geodetic equipment should be evaluated especially in cases where high accuracy is required. Their reliable estimation by methods of modern statistics is subject to sufficient redundancy of measurements, which is in cases of local geodetic networks generally fulfilled. Accuracy characteristics of the total station ELTA 3 were estimated by the method MINQUE (Minimum Norm Quadratic Unbiased Estimation). MINQUE is the parent method for some other method of 2nd order parameter estimations, one of which Best Invariant Quadratic Unbiased Estimation (BIQUE) and Marginal Maximum Likehood Estimation (MMLE) are often cited in books of modern statistics (Chen, 1990; Caspary, 1987; Gašincová, 2007).

Stochastic part of the regular Gauss-Markoff model (1) is prescribed in the form:

Tačnost ocenjenih koordinata mreže, pojedinačna gradnja različitih brojčanih pokazatelja tačnosti i k – interval poverenja velikih područja određen je iz kovarijans matrice geodetskih zadataka (Šíma, 2008; Sabová, Gašincová, 2002; Lipták, 2009)

$$\mathbf{\Sigma}_{\hat{\Theta}} = s_0^2(\mathbf{A}^T\mathbf{Q}_1^{-1}\mathbf{A})^{-1} = s_0^2\mathbf{Q}_{\hat{\Theta}}\,, \qquad (8)$$

Gde je $\boldsymbol{Q}_{\Theta}$ $k\times k$ kofaktor matrica nepoznatih parametara. U smislu tumačenja rezultata izravnanja, ovo se može smestiti u najvažniji ishod postupka procene:

n×n kofaktor matrica ostataka

$$\boldsymbol{Q_v} = \boldsymbol{Q_l} - \boldsymbol{A}(\boldsymbol{A}^T\boldsymbol{Q}_l^{-1}\boldsymbol{A})^{-1}\boldsymbol{A}^T = \boldsymbol{Q_l} - \boldsymbol{A}\boldsymbol{N}^{-1}\boldsymbol{A}^T, \quad (9)$$

$n\times n$ redundantna matrica posmatranih veličina $\boldsymbol{l}$

$$\boldsymbol{R} = (\boldsymbol{Q}_l - \boldsymbol{A}\boldsymbol{N}^{-1}\boldsymbol{A}^T)\boldsymbol{Q}_l^{-1}, \qquad (10)$$

Set izmerenih podataka l u horizontalnoj geodetskoj mreži je kombinacija zemaljskih ugaonih i razdaljinskih merenja izvršenih kod svih stanica, što se korisno može kombinovati sa vektorima izmerenih putem GNSS (GPS) tehnologije. (Gašincová, 2007; Kalatovičová, 2010). Veoma je važno znati, što je moguće bolje, karakteristike preciznosti geodetskih datoteka radi izračunavanja intervala poverenja nepoznatih parametara, tačnog i nepristrasnog statističkog tumačenja rezultata merenja. Procena varijans komponenata geodetske opreme trebalo bi da se izvrši naročito u slučajevima u kojima se zahteva velika tačnost. Njihova pouzdana procena po metodama savremene statistike zavisi od dovoljne redundantnosti merenja, što se obično ispuni u slučajevima lokalnih geodetskih mreža. Karakteristike tačnosti cele stanice ELTA 3 procenjene su metodom MINQUE (Minimum Norm Quadratic Unbiased Estimation- metodom najmanjih kvadrata i minimalne norme). MINQUE je matična (osnovna) metoda za neke druge metode za procene parametara drugog reda, od kojih se Najbolja nepristrasna procena nepromenljivih kvadrata (BIQUE) i Procena marginalne najveće verovatnoće (MMLE) često navode u knjigama o savremenoj statistici (Chen, 1990; Caspary, 1987; Gašincová, 2007).

Stohastički deo pravilnog Gaus-Markovljevog modela (1) napisan je u obliku:

$$\boldsymbol{\Sigma}_l = \sigma_1^2 \boldsymbol{V}_1 + \sigma_2^2 \boldsymbol{V}_2 + \cdots + \sigma_p^2 \boldsymbol{V}_p = \sum_{i=1}^{p} \sigma_i^2 \boldsymbol{V}_i$$
$$= \vartheta_1^0 \boldsymbol{V}_1 + \vartheta_2^0 \boldsymbol{V}_2 + \cdots + \vartheta_p^0 \boldsymbol{V}_p = \sum_{i=1}^{p} \vartheta_i^0 \boldsymbol{V}_i \,, \quad (11)$$

Where $\sigma_i>0$, $i=1,2, \ldots p$. ϑ_i^0 are estimates or a priori variance components, $\boldsymbol{V}_i$ the corresponding positive semidefinitné matrix n-th order. Impartial invariant, quadratic estimate (MINQUE) with minimum norm parameter 2nd order $(\hat{\vartheta}_1, \cdots, \hat{\vartheta}_p)^T$ is given by

$$\underbrace{\begin{bmatrix} tr(\boldsymbol{MV}_1\boldsymbol{MV}_1) & \cdots & tr(\boldsymbol{MV}_1\boldsymbol{MV}_p) \\ & \vdots & \\ tr(\boldsymbol{MV}_p\boldsymbol{MV}_1) & \cdots & tr(\boldsymbol{MV}_p\boldsymbol{MV}_p) \end{bmatrix}}_{S} \begin{bmatrix} \hat{\vartheta}_1 \\ \vdots \\ \hat{\vartheta}_p \end{bmatrix} = \underbrace{\begin{bmatrix} \boldsymbol{l}^T\boldsymbol{MV}_1\boldsymbol{Ml} \\ \vdots \\ \boldsymbol{l}^T\boldsymbol{MV}_p\boldsymbol{Ml} \end{bmatrix}}_{q}, \quad (12)$$

where matrix $\boldsymbol{M}$ is calculated according to equation

$$\boldsymbol{M} = \boldsymbol{\Sigma}_l^{-1} - \boldsymbol{\Sigma}_l^{-1}\boldsymbol{A}(\boldsymbol{A}^T\boldsymbol{\Sigma}_l^{-1}\boldsymbol{A})^{-1}\boldsymbol{A}^T\boldsymbol{\Sigma}_l^{-1}. \quad (13)$$

Estimated coefficients of linear combination $(\hat{\vartheta}_1, \cdots, \hat{\vartheta}_p)^T$ are determined by iterative calculation from the relationships (11) a (12). Iterative cycle stops when it is satisfied condition $\left|\hat{\vartheta}_i - \vartheta_i\right| \le \delta$ for all *i*. δ is predetermined constant from field of real numbers that defines the degree of accuracy of estimated parameters.

Statistical testing of blunders that entered into set of measured values is performed (Caspary, 1987; Ghilani 2006):

- By global test of model (1)

$$\frac{\boldsymbol{v}^T\boldsymbol{Q}_l^{-1}\boldsymbol{v}}{\delta_0^2} = \frac{(n-k)s_0^2}{\delta_0^2} \approx \chi_\alpha^2(n-k) \quad (14)$$

- By localization test of blunders from which may mentioned for example Pope's Tau Method

$$\frac{|v_i|}{s_0\sqrt{q_{vi}}} \approx \tau_\alpha(n-k) \quad (15)$$

If the tests (14) and (15) prove presence of outlying measurements in $\boldsymbol{l}$, it is possible to reduce the impact of these measurements by

Gde su $\sigma_i>0$, $i=1,2, \ldots p$. ϑ_i^0 procene ili apriori varijans komponente, $\boldsymbol{V}_i$ odgovarajuća pozitivna polukonačna matrica n-tog reda. Nepristrasna nevarijabilna, kvadratna procena (MINQUE) sa parametrom drugog reda minimalne norme $(\hat{\vartheta}_1, \cdots, \hat{\vartheta}_p)^T$ data je putem

gde je matrica M izračunata prema jednačini

Procenjeni koeficijenti linearne kombinacije $(\hat{\vartheta}_1, \cdots, \hat{\vartheta}_p)^T$ određuju se putem ponovnog izračunavanja iz relacija (11) i (12). Iterativni ciklus prestaje kada je ispunjen uslov $\left|\hat{\vartheta}_i - \vartheta_i\right| \le \delta$ za sve i. δ je unapred određena konstanta od polja realnih brojeva, koja definiše stepen tačnosti procenjenih parametara.

Vrši se statističko testiranje grubih greški koje je uneto u set izmerenih vrednosti (Caspary, 1987; Ghilani 2006):

- Putem globalnog testiranja modela (1)

- Putem lokalnog testiranja greški od kojih možemo pomenuti na primer Poupovu Tau metodu

Ukoliko ispitivanja (14) i (15) dokažu prisustvo izbačenih merenja u l, moguće je umanjiti uticaj ovih merenja grubim uravnanjem. Primećujemo robust adjustment. It may be noted that primarily the Danish method is successfully used in geodetic practice. Principle of the method is that large residuals indicate less accurate observations and vice-versa (Gašincová, 2007; Caspary, 1987). After least squares convention estimates of the model (1), the a priori weights are replaced by new ones being functions of residuals (17). The original weighs of measurement are overwrite in the next iteration step by the relationship

$$p_{i+1} = p_i f(v_i)\text{, i= 1,2, ...,} \tag{16}$$

where

$$f(v_i) = \begin{cases} 1 & pre \ \frac{|v_i|\sqrt{p_1}}{s_0} < c \\ exp\left(-\frac{|v_i|p_1}{cs_0}\right) & pre \ \frac{|v_i|\sqrt{p_1}}{s_0} > c \end{cases} \tag{17}$$

The constant c is usually selected between 2 and 3. Iteration cycle is repeated until desired results are not met, and usually does not exceed 10-15 iterations.

da se u većini slučajeva uspešno koristi Danska metoda u geodetskoj praksi. Princip ove metode je da veliki ostaci ukazuju na to da su opažanja manje tačna i obrnuto (Gašincová, 2007; Caspary, 1987). Posle procena modela po metodi najmanjih kvadrata (1), apriori mere se zamenjuju novim budući da su one funkcije ostataka. Prvobitne mere su poništene u sledećem iterativnom koraku putem relacije

$$p_{i+1} = p_i f(v_i)\text{, i= 1,2, ...,} \tag{16}$$

gde je

$$f(v_i) = \begin{cases} 1 & pre \ \frac{|v_i|\sqrt{p_1}}{s_0} < c \\ exp\left(-\frac{|v_i|p_1}{cs_0}\right) & pre \ \frac{|v_i|\sqrt{p_1}}{s_0} > c \end{cases} \tag{17}$$

Konstanta c se obično bira između 2 i 3. Iterativni ciklus se ponavlja dok se ne dobiju željeni rezultati, a broj ponavljanja obično ne prelazi 10-15.

4 EMPIRICAL DEMONSTRATION

The above mathematical relations were applied to the processing of horizontal setting out network of highway bridge object 207-00 "Jánošíková studnička".

Table 1 compares the results obtained by least squares method (LSM) of the model (1) and by the Danish method.

4 EMPIRIJSKA DEMONSTRACIJA

Gore navedene matematičke relacije su primenjene na horizontalno uspostavljanje mreže objekta - mosta na autoputu 207-00 "Jánošíková studnička".

U Tabeli 1 upoređeni su rezultati dobijeni metodom najmanjih kvadrata (LSM) modela (1) i Danskom metodom.

Table 1 The comparation of results of estimation obtained by Danish method and by LSM
Tabela 1 Poređenje rezultata procene dobijene Danskom metodom i metodom najmanjih kvadrata

point	LSM				Danish method			
	$\hat{X}$ [m]	$\hat{Y}$ [m]	$s_{\hat{X}\hat{Y}}$ [mm]	S_P [mm]	$\hat{X}$ [m]	$\hat{Y}$ [m]	$s_{\hat{X}\hat{Y}}$ [mm]	S_P [mm]
2071	545.5051	2360.5917	0.92	1.30	545.5043	2360.5914	0.65	0.92
2072	500.0000	2360.0630	0.59	0.84	500.0000	2360.0627	0.41	0.58
2073	554.6821	2146.4392	0.88	1.24	554.6818	2146.4394	0.60	0.85
2074	498.2166	2181.9492	1.06	1.50	498.2167	2181.9485	0.73	1.03
2075	550.2718	2011.6291	0.79	1.11	550.2714	2011.6291	0.51	0.72
2076	500.0000	2000.0000	0.00	0.00	500.0000	2000.0000	0.00	0.00
1171	593.3287	2166.2131	1.21	1.71	593.3271	2166.2127	0.87	1.23
5001	551.9829	1993.4855	0.81	1.14	551.9825	1993.4851	0.54	0.76
1160	543.9166	1876.8222	1.64	2.32	543.9158	1876.8217	1.07	1.52
1172	722.1130	2024.7659	1.75	2.47	722.1123	2024.7655	1.15	1.63
6901	481.5995	2354.0258	1.05	1.49	481.5997	2354.0256	0.73	1.03
σ_d:	1.6 [mm]				σ_d: 1.0 [mm]			
σ_ψ:	6.0 [cc]				σ_ψ: 3.9 [cc]			

$\hat{X},\hat{Y}$ - Adjustment coordinates;

σ_d, σ_ψ - MINQUE estimates of standard deviations of the measured lengths and directions;

$s_{\hat{X}\hat{Y}}$ - The mean coordinate error;

$$s_{\hat{X}\hat{Y}} = \sqrt{\frac{s_{\hat{X}}^2 + s_{\hat{Y}}^2}{2}} \qquad (18)$$

S_p The mean positional error;

$$s_P = \sqrt{s_{\hat{X}}^2 + s_{\hat{Y}}^2}\,. \qquad (19)$$

Graphic representation of median positional error in any direction Θ is shown in Figure 1

$$s_\Theta^2 = \lambda_1 \cos^2\Theta + \lambda_2 \sin^2\Theta\,, \qquad (20)$$

λ_1, λ_2 are eigenvalues of covariance matrix (8) of random vector $[\hat{X},\hat{Y}]^T$

$\hat{X},\hat{Y}$ Koordinate uravnanja;

σ_d, σ_ψ - MINQUE procene standardnih odstupanja izmerenih dužina i pravaca;

$s_{\hat{X}\hat{Y}}$ - Srednja koordinantna greška;

$$s_{\hat{X}\hat{Y}} = \sqrt{\frac{s_{\hat{X}}^2 + s_{\hat{Y}}^2}{2}} \qquad (18)$$

S_p Sredina poziciona greška;

$$s_P = \sqrt{s_{\hat{X}}^2 + s_{\hat{Y}}^2}\,. \qquad (19)$$

Na slici 1 nalazi se grafički prikaz srednje pozicione greške u bilo kom pravcu Θ .

$$s_\Theta^2 = \lambda_1 \cos^2\Theta + \lambda_2 \sin^2\Theta\,, \qquad (20)$$

λ_1, λ_2 su svojstvene vrednosti kovarijans matrice (8) i nasumičnog vektora $[\hat{X},\hat{Y}]^T$

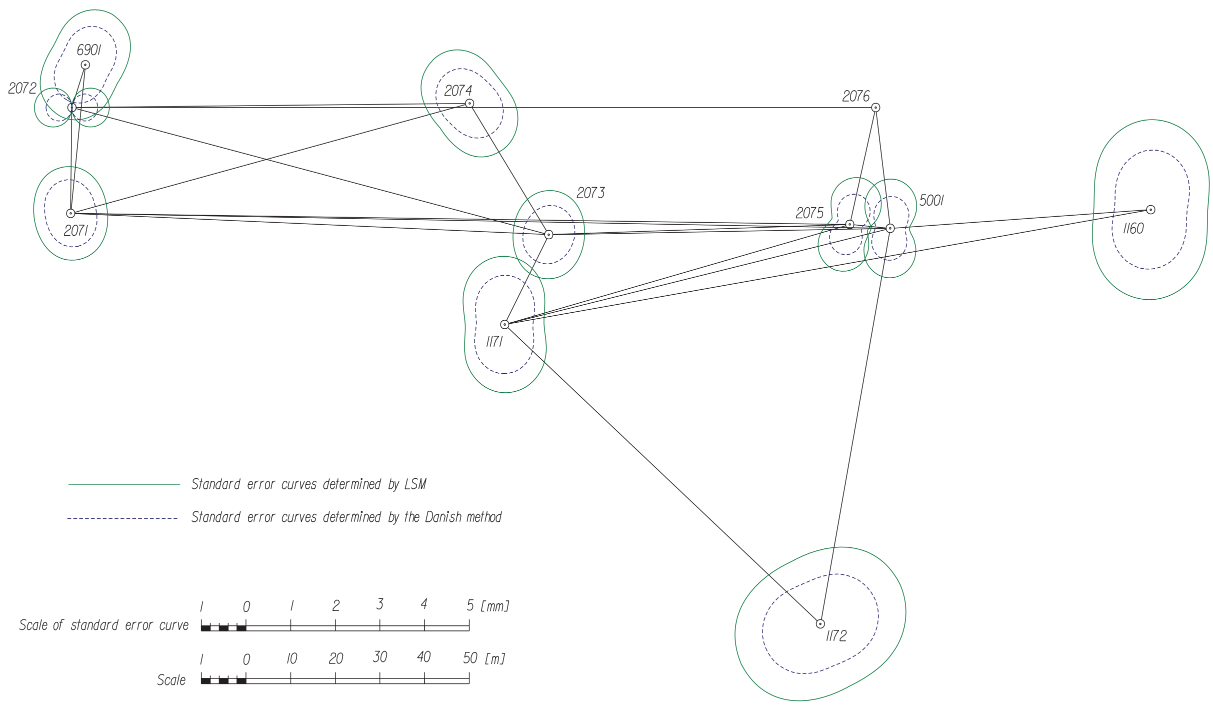

Figure 1 of horizontal setting out network of highway bridge object 207-00 "Jánošíková studnička"
slika 1 horizontalno uspostavljanje objekta-mosta na autoputu 207-00 "Jánošíková studnička"

5 CONCLUSION

Reliable setting out network is an essential prerequisite for meeting the high standards of accuracy of geodetic works realized for continually technologically difficult, longer and higher bridge objects Land surveyor can fulfill these high demands

5 ZAKLJUČAK

Pouzdano uspostavljanje mreže je neophodan preduslov za ispunjavanje visokih standarda tačnosti geodetskih radova izvršenih za kontinualno-tehnološki teške, dugačke i visoke mostne objekte. Geometar može ispuniti ove through the accurate and powerful measuring and computing techniques. Especially computer technology allows the use of exact mathematical methods and procedures that in light of large volume of numerical calculations in practical geodesy hardly find their application. Presented results were obtained in real conditions and demonstrated the validity of the use of new mathematical methods with aim get more objective and accuracy results.

visoke zahteve putem tačnog i jasnog merenja i tehnika računanja. Naročito kompjuterska tehnologija omogućava korišćenje tačnih matematičkih metoda i postupaka koji, u svetlu obimnih brojčanih izračunavanja, u geodeziji retko nalaze svoju primenu. Prikazani rezultati su dobijeni u realnim uslovima i pokazuju valjanost korišćenja novih matematičkih metoda sa ciljem da se dobiju objektivniji i tačniji rezultati.

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