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THE INFLUENCE OF USING THE DIFFERENT FORECASTING TYPES TO THE ACCURACY OF THE FORECASTED PRICES OF THE COMMODITY

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Abstract: *In mathematical models forecasting the prices on the commodity exchanges statistical methods are usually used. In the paper we derive the numerical model based on the exponential approximation of the stock exchanges of the commodity. The price prognoses of Aluminium on the London Metal Exchange are determined as the numerical solution of the Cauchy initial problem for the 1st order ordinary differential equation in the form* $y' = a_1 y, \ \ y(x_0) = y_0$. *To solve the considered type of the Cauchy initial problem, the embedded Runge-Kutta formulae of the 5th order to the 6th order is used. When forecasting monthly average prices, we compare the accuracy of the prognoses acquired either in direct way (monthly forecasting) or as the arithmetic mean of the daily prognoses (daily forecasting). The advantages of the studied types of forecasting during different movements of the Aluminium prices and using the different lengths of the approximation terms are analyzed.*

Key words: *Forecasting, Numerical modelling, Exponential approximation, The Cauchy initial problem for the ordinary differential equation*

1 INTRODUCTION

Observing trends and forecasting the movements of metal prices is still a current problem. There are a lot of approaches to forecasting the price movements. Some of them are based on mathematical models. Forecasting the prices on the commodity exchanges often uses the statistical methods [4], [5] that need to process a large number of the historical market data. The amount of the needed market data can sometimes be a disadvantage. In such cases other mathematical methods are required.

We have decided to use the numerical methods. Their advantage is that much less market data is needed in comparison with the statistical models. Our numerical model for forecasting prices is based on the numerical solution of the Cauchy initial problem for the 1st order ordinary differential equations [3].

In our prognostic model we came out of the Aluminium prices presented on the London Metal Exchange (LME) [6]. We dealt with the monthly averages of the daily closing Aluminium prices "Cash Seller&Settlement price" in the period from December 2002 to June 2006. We obtained the market data from the official web page of the London Metal Exchange [6]. The course of the Aluminium prices on LME (in US $ per tonne) in the observing period is presented in Figure 1.

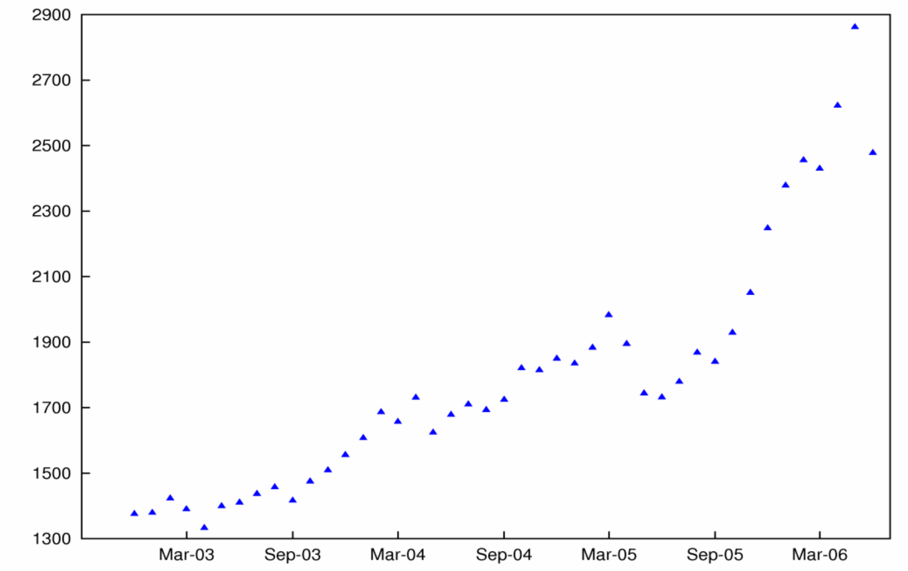

Fig. 1 *The course of the Aluminium prices on LME in years 2003 - 2006*

As we can see in Figure 1 the course of the Aluminium prices in the considered period changes dramatically.

2 MATHEMATICAL MODEL

We shall consider the Cauchy initial problem in the form

$$y' = a_1 y, \;\; y(x_0) = y_0 . \qquad (1)$$

The particular solution of the problem (1) is in the form $y = k\, e^{a_1 x}$, where $k = y_0\, e^{-a_1 x_0}$. The considered exponential trend was chosen according to the basis of the test criterion of the time series' trend suitability. The values $\ln(Y_{i+1}) - \ln(Y_i)$, for $i = 0,1,\dots,42$ have approximately constant course. (Y_i is the Aluminium price (stock exchange) on LME in the month x_i.)

The price prognoses are created by the following steps:

The 1st step: Approximation of the values – the values of the approximation term are approximated by the least squares method. The exponential function in the form $\tilde{y} = a_0 e^{a_1 x}$ is used. We create the approximation terms of the different lengths. Let's consider 4 different variants:

Variant A: Three approximation terms of the different length are made. They are determined by the stock exchanges in the year 2005; in the years 2004 and 2005; in the years 2003, 2004 and 2005 (Figure 2).

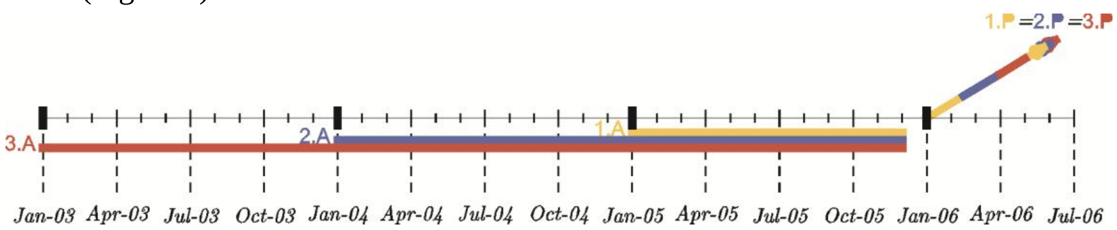

Fig. 2 *Variant A (A – approximation term, P – forecasting term)*

Variant B: Values from the period January 2003 - June 2003 are approximated. The next approximation terms are created by sequential extension of this period by 3 months. Thus the duration of the approximation terms is extended (the n^{th} approximation term has $6+3(n-1)$ stock exchanges) (Figure 3).

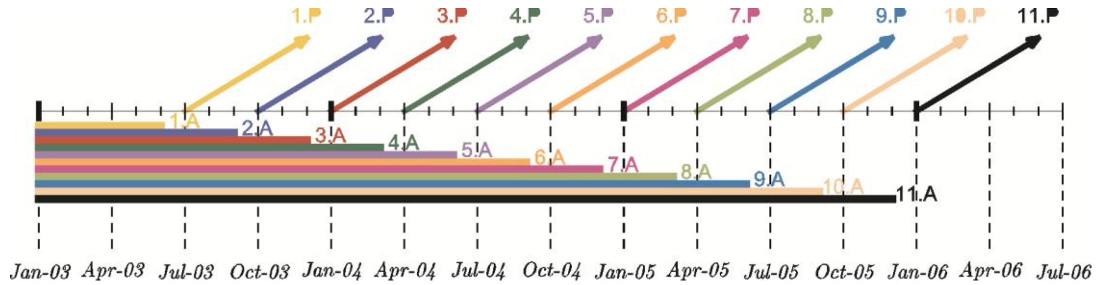

Fig. 3 *Variant B (A – approximation term, P – forecasting term)*

Variant C: We approximate the values of 12 months and each term is shifted by 1 month (Figure 4). (The first approximation term is January 2003 - December 2003.)

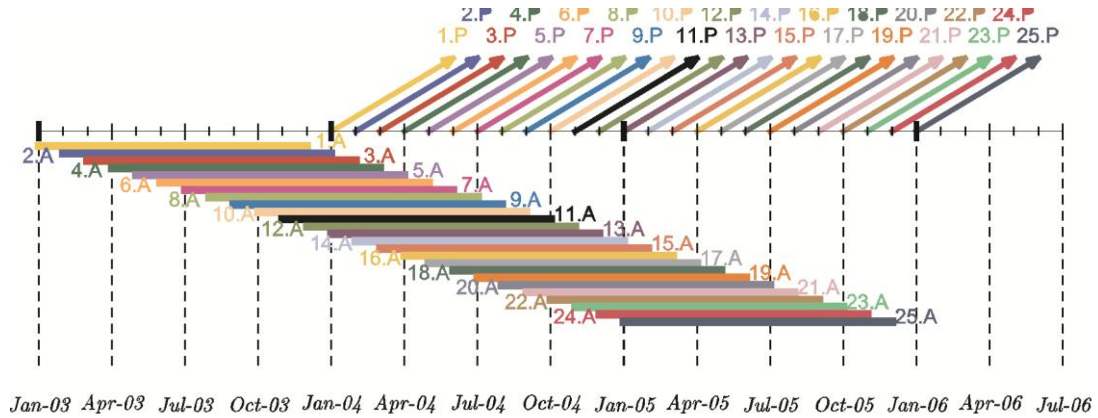

Fig. 4 *Variant C (A – approximation term, P – forecasting term)*

Variant D: We approximate the values of 6 months shifting these periods forward by 3 months (Figure 5). (The first approximation term is January 2003 - June 2003.)

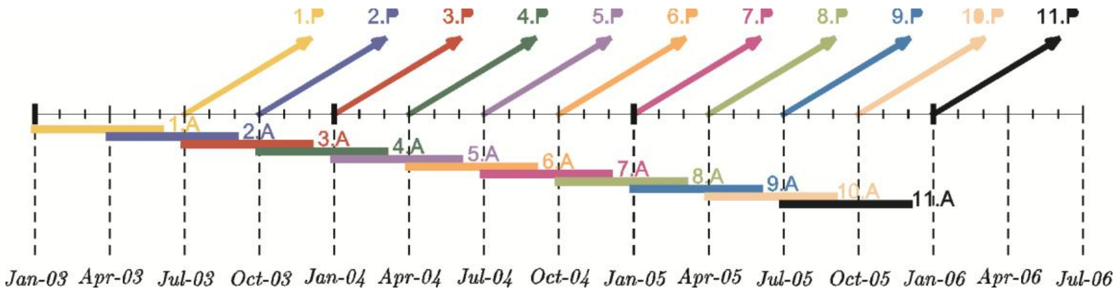

Fig. 5 *Variant D (A – approximation term, P – forecasting term)*

The 2nd step: Formulating the Cauchy initial problem – according to the acquired approximation function $\tilde{y}$ the Cauchy initial problem (1) is written in the form

$$y' = a_1 y, \ \ y(x_i) = Y_i, \qquad (2)$$

where $x_i = i$ and Y_i is the Aluminium price on LME in the month x_i, which is the last month of the approximation term.

The 3rd step: Computing the prognoses – the formulated Cauchy initial problem (2) is solved by the numerical method – the embedded Runge-Kutta formulae of the 5th order to the 6th order. The method uses the following numerical formulae [3]:

$$x_{i+1} = x_i + h,$$

$$y_{i+1} = y_i + \frac{1}{144}\left(9K_i^{[1]} + 40K_i^{[3]} + 20K_i^{[4]} + 30K_i^{[5]} + 35K_i^{[6]} + 10K_i^{[7]}\right),$$

$$\hat{y}_{i+1} = \hat{y}_i + \frac{1}{288}\left(19K_i^{[1]} + 75K_i^{[3]} + 50K_i^{[4]} + 50K_i^{[5]} + 75K_i^{[6]} - 9K_i^{[7]} + 28K_i^{[8]}\right),$$

where

$$K_i^{[1]} = h_i \ f(x_i, y_i),$$

$$K_i^{[2]} = h_i \ f(x_i + \frac{1}{10}h_i, y_i + \frac{1}{10}K_i^{[1]}),$$

$$K_i^{[3]} = h_i \ f(x_i + \frac{1}{5}h_i, y_i + \frac{1}{5}K_i^{[2]}),$$

$$K_i^{[4]} = h_i \ f(x_i + \frac{2}{5}h_i, y_i - \frac{1}{5}K_i^{[1]} + \frac{2}{5}K_i^{[2]} + \frac{1}{5}K_i^{[3]}),$$

$$K_i^{[5]} = h_i \ f(x_i + \frac{3}{5}h_i, y_i + \frac{31}{30}K_i^{[1]} - \frac{64}{30}K_i^{[2]} + \frac{43}{30}K_i^{[3]} + \frac{8}{30}K_i^{[4]}),$$

$$K_i^{[6]} = h_i \ f(x_i + \frac{4}{5}h_i, y_i + \frac{2}{35}K_i^{[1]} + \frac{20}{35}K_i^{[2]} + \frac{3}{35}K_i^{[3]} - \frac{34}{35}K_i^{[4]} + \frac{37}{35}K_i^{[5]}),$$

$$K_i^{[7]} = h_i \ f(x_i + h_i, y_i - \frac{20}{10}K_i^{[1]} + \frac{28}{10}K_i^{[2]} - \frac{18}{10}K_i^{[3]} + \frac{38}{10}K_i^{[4]} - \frac{25}{10}K_i^{[5]} + \frac{7}{10}K_i^{[6]}),$$

$$K_i^{[8]} = h_i \ f(x_i + h_i, y_i - \frac{9440}{5880}K_i^{[1]} + \frac{11342}{5880}K_i^{[2]} - \frac{9302}{5880}K_i^{[3]} + \frac{25982}{5880}K_i^{[4]} - \frac{17175}{5880}K_i^{[5]} + \frac{4473}{5880}K_i^{[6]}),$$

for $i = 1, 2, 3, \ldots$, where $h = x_{i+1} - x_i$ is the step of the constant size.

Within the observed period the differential equations of the Cauchy initial problem in the form $y' = a_1 y$, where $a_1 < 0{,}05$, are created. Because the values of the coefficient a_1 are too low, the same results of numerical solutions y_i and $\hat{y}_i$ are acquired. Thus we didn't obtain the interval of the solution $\langle y_i, \hat{y}_i \rangle$ [1], but the only value $y_i = \hat{y}_i$. To acquire the interval of the solution $\langle y_i, \hat{y}_i \rangle$, the coefficient a_1 had to obtain value at least $0{,}26$ in monthly forecasting and $0{,}42$ in daily forecasting.

Computed value y_{i+1} is considered as the Aluminium price prognosis in the month x_{i+1}. The prognoses are computed by two types of forecasting:

1. **monthly forecasting**
   Using the values $[x_i, Y_i]$ we directly compute the numerical solution $[x_{i+1}, y_{i+1}]$ by solving the Cauchy initial problem (2).
2. **daily forecasting**
   In this case the prognosis y_{i+1} in the month x_{i+1} is obtained from values $[x_i, Y_i]$ by using more partial computations.

   The interval $\langle x_i, x_{i+1} \rangle$ of the length $h = 1$ month is divided into n parts, where n is the number of trading days on LME in the month x_{i+1} [6]. We gain the sequence of the points $x_{i0} = x_i$, $x_{ij} = x_i + \frac{h}{n} j$, for $j = 1, 2, ..., n$, where $x_{in} = x_{i+1}$. For each point of the subdivision of the interval, the Cauchy initial problem in the form (2) is solved by chosen numerical method. In this way we obtain the prognoses of the Aluminium prices on trading days y_{ij}. By computing the arithmetic mean of the daily prognoses we obtain the monthly prognosis of the Aluminium price in the month x_{i+1}. Thus, $y_{i+1} = \frac{\sum_{j=1}^{n} y_{ij}}{n}$.

We calculate the prognoses in six months after the end of the approximation term. The prognoses y_{i+s}, where $s = 2, 3, 4, 5, 6$ are obtained after modification of the initial condition value. The initial condition value in the month x_{i+s}, $s = 1, 2, 3, 4, 5$ is replaced by calculated prognosis y_{i+s}. The prognosis y_{i+s+1}, $s = 1, 2, 3, 4, 5$ is gained by solving the Cauchy initial problem $y' = a_1 y,\ y(x_{i+s}) = y_{i+s}$.

The calculated prognosis y_s in the month x_s is compared with the real stock exchange Y_s. We evaluate the absolute percentage error $|p_s| = \frac{|y_s - Y_s|}{Y_s} .100\,\%$. The prognosis y_s in the month x_s is acceptable in practice, if $|p_s| < 10\,\%$. Otherwise, it is called the critical value. To compare the accuracy of forecasting of all forecasting terms, the mean absolute percentage error (MAPE) $\bar{p} = \frac{\sum_{s=1}^{t} |p_s|}{t}$ is determined [5], where, in our case, $t = 6$.

3 COMPARING THE RESULTS OF BOTH MONTHLY FORECASTING AND DAILY FORECASTING

3.1 Variant A

In variant A, monthly forecasting is more accurate in the forecasting term *January 2006 – June 2006* for all determined approximation terms. Comparing the values of both types of forecasting and the Aluminium stock exchange in observing months we have found out that the daily forecasted prognoses are lower when the price increases and they are higher when the price decreases in comparison with the prognoses obtained by monthly forecasting. Thereby neither the increase nor the decrease of the daily forecasted prognosis is large. Therefore daily forecasting can be considered slighter than monthly forecasting.

Because the increase of the prices in the forecasting term *January 2006 – June 2006* is rapid, the daily forecasted prognoses follow this course of the prices not enough, so they are less accurate. Obtained results of forecasting are summarized in Table 1.

Tab. 1 *The comparison of the success of determined types of forecasting - variant A [MAPE in %]*

Approximation term	Monthly forecasting	Daily forecasting
year 2005	7,92	9,43
years 2004, 2005	8,33	9,65
years 2003, 2004, 2005	7,47	9,20

3.2 Variant B

In this variant we obtain 11 forecasting terms with the results in Table 2.

Tab. 2 *The comparison of the success of determined types of forecasting - variant B*

Criterion	Monthly forecasting	Daily forecasting
The arithmetic mean of MAPE	6,82 %	7,20 %
The number of more accurate price prognoses	8	3
The number of the critical values	18	17
The number of MAPE ≥ 10 %	2	2

Monthly forecasting is more accurate. The lower arithmetic mean of MAPE and more often more accurate results in observed forecasting terms are pointed at this fact. The number of the critical values is similar in the both types of forecasting. Daily forecasting is more accurate only in 3 forecasting terms. Larger decline of the Aluminium prices is typical for these terms. The increase of the daily forecasted prognoses is smaller than the increase of the prognoses obtained by monthly forecasting. Thus, the daily forecasted prognoses are closer to the real stock exchanges, which are fallen. At the stable increasing course of the prices the higher increase of the monthly forecasted prognoses is more advantageous.

3.3 Variant C

Shortening the length of the approximation term, the approximation functions respond to the course of the Aluminium prices markedly. Higher variability of the course of the exponential functions makes more successful moderate increasing or decreasing prognoses. That's typical for the prognoses of daily forecasting.

The lower modification of the daily forecasted prognoses is more advantageous in the following periods of the course of the commodity price:

a) the price decline after the increasing price course,
b) the moderate price increase after more rapid price increase within the approximation term,
c) the increase of the stock exchanges when the approximation function has decreasing course.

The advantage of the slower increasing daily forecasted prognoses is that they are closer to the real stock exchanges, which are fallen (a) or increased slower than the stock exchanges in the approximation term (b). In case of the decreasing values of the prognoses (the course of the approximation function is decreasing) the moderate decrease of the daily forecasted prognoses is more advantageous to forecast the increasing stock exchanges. If the intensity of the increase of the stock exchanges after the increasing price course in the approximation term is kept, the larger increase of the monthly forecasted prognoses is more advantageous. The results of forecasting for 25 forecasting terms determined in this variant are shown in Table 3.

Tab. 3 *The comparison of the success of determined types of forecasting - variant C*

Criterion	Monthly forecasting	Daily forecasting
The arithmetic mean of MAPE	7,40 %	7,18 %
The number of more accurate price prognoses	13	12
The number of the critical values	44	35
The number of MAPE ≥ 10 %	7	6

3.4 Variant D

In this variant 11 forecasting terms are created. The results of forecasting types are in the following table.

Tab. 4 *The comparison of the success of determined types of forecasting - variant D*

Criterion	Monthly forecasting	Daily forecasting
The arithmetic mean of MAPE	8,36 %	7,70 %
The number of more accurate price prognoses	7	4
The number of the critical values	21	19
The number of MAPE ≥ 10 %	3	3

There are the shortest approximation terms in variant D. Because only 6 values are approximated, the course of the approximation function in the exponential form can be quicker changed in comparison with approximating the higher number of the values [2]. Because the changes of the approximation functions are rapid, so the rapid is also the course of the calculated prognoses. Thereby the slower increase or decrease of the daily forecasted prognoses is more advantageous.

4 CONCLUSIONS

Comparing the results of two determined types of forecasting (daily and monthly forecasting) by using the embedded Runge-Kutta formulae of the 5th order to the 6th order, the differences are little. In variants using longer approximation terms (variant A and B) monthly forecasting is more accurate. The monthly forecasted prognoses are rapidly increasing/decreasing than the daily forecasted prognoses. In variants using shorter approximation terms (variant C and D) daily forecasting, with slower changing values of the prognoses, is more advantageous. Using longer approximation terms the approximation functions are more moderate, so the rapid increase of the monthly forecasted prognoses is more advantageous. On the other hand, shorter approximation terms much more rapidly response also small changes, thereby the course of the approximation functions is more rapid, so the slower increase of the daily forecasted prognoses is more advantageous.

In regard to rate of the increase or the decrease of the prognoses, the rapid change of the monthly forecasted prognoses is more advantageous at the stable price course. Otherwise, the slower change of the daily forecasted prognoses provides more accurate forecasting during the changes at the price course or when the increase or the decrease of the stock exchanges gets slower.

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