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MODEL OF CONTAINER LINE CIRCULAR ROUTE OPTIMIZATION

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Abstract: *A model of selecting optimal container line route according to the criterion of maximal profit intensity has been suggested. The model generalizes the travelling salesman problem and permits to exclude unprofitable ports from the route and provides the necessary balance of laden and empty containers. Separate records of route, cargo and time variables enable to combine finance and time factors within a model.*

Key words: *optimization model, container line route, container, profit intensity, travelling salesman problem, port.*

1 INTRODUCTION

Nowadays among the main trends in sea transport development a considerable place is taken by line shipping and container transportations in particular. Competition for customers on the world market dictates special conditions of work for container lines. Thus, high cost of cargo that is transported in line shipping sets up strict requirements to the speed and regularity of its delivery.

Above circumstances cause the necessity of science-based management of transportation process, which would provide reliability of transport services besides direct rise of transportation effectiveness.

From the position of container line operator transportation effectiveness could be raised by elaboration of optimal vessel route among ports, each of which has certain freight flow.

Scientific papers [1–3] deal with problems of routes optimization at line or container transportations. However, these papers were written under the conditions of planned economy and can not be used under current conditions without appropriate changes.

Thus, paper [1] suggests a model of selecting optimal vessel route in line shipping, which can be used mostly for bulk cargo (but taking into account that vessel may be loaded with containers). Besides, some requests for transportation (connected with the state plan) should be obligatorily executed, that can not be applied to market economy, when a ship-owner is able to select the most profitable cargo.

Scientific papers [2] and [3], that define optimal plans of line shipping companies, suggest some mathematical models for solution of the above stated problem. Besides, the mentioned models need the availability of preselected vessel routes. Such simplicity does not always permit to reproduce the essence of transportation process adequately. Sphere of application of these models is limited by lines with small amount of ports of call.
Thus, the elaboration of economic and mathematical model is necessary. This model has to find optimal container line route subject to market economy principles and peculiarities, typical for container transportations on current stage of their development.

2 TRAVELLING SALESMAN PROBLEM IN CONTAINER LINE ROUTE OPTIMIZATION

The solution of this problem is a complex of interdependent tasks, which concern not only the determination of vessel traffic routes relative to planned cargo traffic, but also managing the empty containers pool, including redistribution of the pool among ports. As redistribution of empty containers is directly connected with their transportation, the optimization of vessel traffic route should be resolved from the position of connecting empty containers flow and cargo flow (loaded containers flow) into one traffic scheme [1].

It is suggested to take classic travelling salesman problem (TSP) as a basis. This problem consists in finding the most profitable route that visits all determined points (ports) exactly once and then returns to the original point (port) [4]. Such situation is shown at Fig. 1.

Travelling salesman problem permits to select different criteria of route profitability (the shortest, the cheapest, etc.).

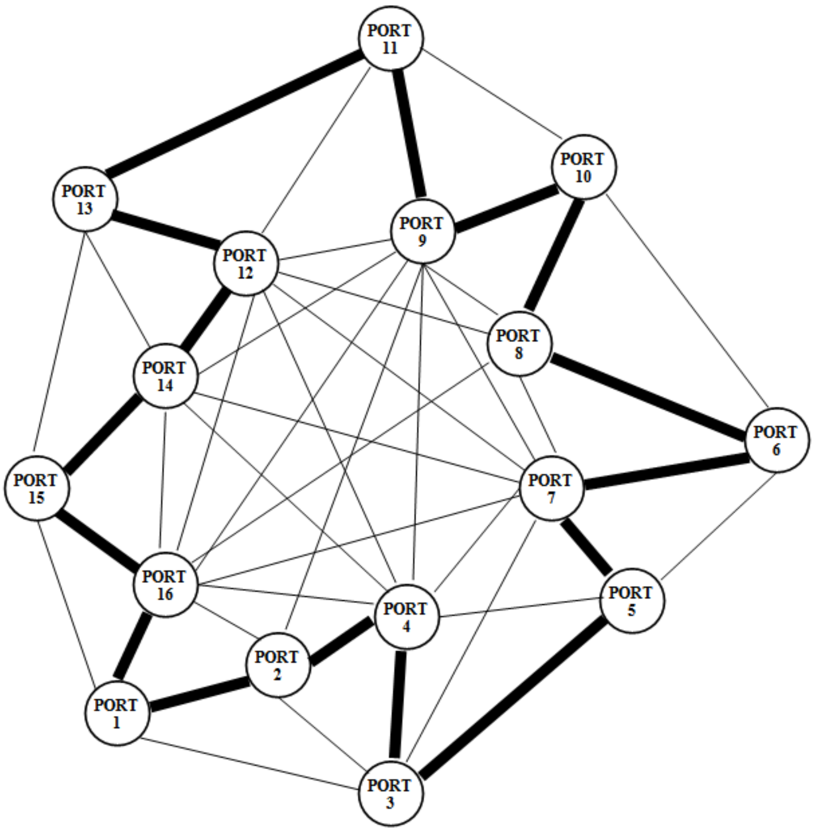

Fig. 1. Possible circular vessel route

Concerning optimization of container line route, it is suggested to use criterion of maximal profit intensity, i.e. gaining profit per a time unit. This criterion is the most acceptable for private shipping companies, which operate in competition conditions on the market of container transportations, because it permits to join the factors of expenses, profits and time into a one function.

Transition to the criterion of maximal profit intensity makes it possible to withdraw the obligatory requirement to call the each port of the route [5]. At usual minimization criteria such withdrawal would lead to total inactivity in optimal plan. Thus, there appears an opportunity to choose from total amount of ports only most suitable ports, among which the route will be optimized.

3 MODEL FOR CIRCULAR ROUTE OPTIMIZATION

Let a container shipping company is searching for an optimal circular route among *n* ports. In this case:

Q_{ij} – average flow of laden containers from port *i* to port *j*, which is formed in a unit of time, $i, j = \overline{1, n}$, TEU/per day;

F_{ij} – freight rate for 1 TEU from port *i* to port *j*, $i, j = \overline{1, n}$, USD/TEU;

t_{ij} – vessel transit time from port *i* to port *j* (vessel might not transit from port *i* to port *j* directly, but through a chain of neighbor ports in other time), $i, j = \overline{1, n}$, days.

Let us enter Boolean variables

$$x_{ij} = \begin{cases} 1, & \text{if there is a direct transfer from port } i \text{ to port } j, \\ 0 & \text{– in opposite case.} \end{cases} \quad (1)$$

$$y_i = \begin{cases} 1, & \text{if port } i \text{ is included in route,} \\ 0 & \text{– in opposite case.} \end{cases} \quad (2)$$

Shipping company profits (*D*, USD) from operation on a route are formed of freight rate for laden containers transportation, multiplied by their number (the company does not gain income from transportation of empty containers):

$$D = \sum_{i=1}^{n} \sum_{j=1}^{n} F_{ij} Z_{ij} y_i y_j \,, \quad (3)$$

where Z_{ij} – is the number of laden containers, transported from port *i* to port *j*, $i, j = \overline{1, n}$, TEU.

Variables y_i and y_j permit to "nullify" possible profits in case if any of the ports *i* or *j* is not included into the route scheme.

Expenses of shipping line (*R*, USD) can be described as follows:

$$R = \sum_{i=1}^{n} p_i^{'} \cdot y_i \cdot (\sum_{j=1}^{n} Z_{ij} + z_{ij}) + \sum_{i=1}^{n} p_i^{''} \cdot y_i \cdot (\sum_{j=1}^{n} Z_{ji} + z_{ji}) + \sum_{i=1}^{n}\sum_{j=1}^{n} R_{ij} \cdot x_{ij} + + \sum_{i=1}^{n} c_i^{'} \cdot y_i + \sum_{i=1}^{n} c_i^{''} \cdot y_i \cdot T_i \cdot \left[\sum_{j=1}^{n} (Z_{ij} + z_{ij}) + \sum_{j=1}^{n} (Z_{ji} + z_{ji}) \right] + b, \quad (4)$$

where $p_i^{'}$ – cost of loading for 1 TEU in port *i*, USD/TEU;
$p_i^{''}$ – cost of unloading for 1 TEU in port *i*, USD/TEU;
z_{ij} – number of empty containers, transported from port *i* to port *j*, $i, j = \overline{1,n}$, TEU;
R_{ij} – cost of direct vessel transfer from port *i* to port *j*, $i, j = \overline{1,n}$, USD;
$c_i^{'}$ – cost of port *i* call, USD;
$c_i^{''}$ – unit cost of mooring at port *i*, USD/day;
T_i – time of loading (unloading) for 1 container in port *i*, days/TEU;
b – fixed vessel expenses, including crew costs, USD;

Similarly, variables y_i, y_j i x_{ij} permit to "nullify" in (4) possible shipping line expenses in case if any of the ports *i* or *j* is not included into route scheme or transfer between them is not direct.

Then the model of optimal container line route selection acquires the following form:

$$I = \frac{D - R}{\sum_{i=1}^{n} T_i \cdot y_i \cdot \left[\sum_{j=1}^{n} (Z_{ij} + z_{ij}) + \sum_{j=1}^{n} (Z_{ji} + z_{ji}) \right] + \sum_{i=1}^{n}\sum_{j=1}^{n} t_{ij} x_{ij} + a} \Rightarrow \max_{\{x_{ij}\},\{Z_{ij}\},\{z_{ij}\}}, \quad (5)$$

where *I* - profit intensity, USD/day,
a – is fixed time expenses (which do not depend on route selection), days.

Sum $\sum_{j=1}^{n} (Z_{ij} + z_{ij})$ at (4) – (5) means number of loaded containers (laden and empty) in port *i*. Similarly, $\sum_{j=1}^{n} (Z_{ji} + z_{ji})$ means number of unloaded containers in port *i*.

Denominator of function (5) describes expenses of time that are necessary for one circular run.

The proposed model has a number of limitations:

$$\sum_{i=1}^{n} x_{ij} = y_j, \ j = \overline{1,n}. \quad (6)$$

$$\sum_{j=1}^{n} x_{ij} = y_i, \ i = \overline{1,n}. \quad (7)$$

$$u_i - u_j + \left(\sum_{i=1}^{n} y_i\right) \cdot x_{ij} \le \left(\sum_{i=1}^{n} y_i\right) - 1, \quad i, j = \overline{1,n}, \qquad (8)$$

where u_i, u_j, $i, j = \overline{1,n}$ – are real numbers, values of which are determined in the process of problem solution [4].

Limitations (6) – (7) provide exactly one call at each selected port of the route and exactly one exit from it.

While in limitations of travelling salesman problem there is a fixed number of ports of route n, in limitation (8), which provides route circularity and absence of "inner circles", there is an unfixed number of ports of call $\sum_{i=1}^{n} y_i \le n$ [5], that are determined in the process of the optimal route scheme problem solution.

It is obvious, that number of fixed for transportation containers in a port should not exceed container flow from this port. Amount of containers that are formed in port grow and accumulate depending on whole time of circular route:

$$Z_{ij} \le Q_{ij} \cdot \left[\sum_{i=1}^{n} T_i \cdot y_i \cdot \left[\sum_{j=1}^{n} (Z_{ij} + z_{ij}) + \sum_{j=1}^{n} (Z_{ji} + z_{ji})\right] + \sum_{i=1}^{n}\sum_{j=1}^{n} t_{ij} x_{ij} + a\right]. \qquad (9)$$

Amount of loaded containers at the moment of entering port j should not exceed container tonnage of vessel:

$$W_j = \sum_{i=1}^{n}\left[W_i + \sum_{l=1}^{n} y_l \cdot (Z_{il} + z_{il}) - \sum_{k=1}^{n} y_k \cdot (Z_{ki} + z_{ki})\right] \cdot x_{ij} \le Q, \qquad (10)$$

where W_j – amount of containers at vessel at the moment of port j entering, TEU;
W_i – amount of containers at vessel at the moment of entering previous port i, TEU;
Q – container tonnage of vessel, TEU.

Limitation (11) provides the selection of the first port, from which circular route will be started:

$$W_1 = \sum_{j=1}^{n} Z_{1j} \le Q. \qquad (11)$$

Similarly, amount of loaded containers on vessel at the moment of leaving port j also should not exceed container tonnage of vessel:

$$V_j = W_j + \sum_{l=1}^{n} y_l \cdot (Z_{jl} + z_{jl}) - \sum_{k=1}^{n} y_k \cdot (Z_{kj} + z_{kj}) \le Q, \qquad (12)$$

where V_j – amount of containers on vessel at the moment of leaving port j, TEU;

If we subtract amount of loaded containers ($\sum_{l=1}^{n} Z_{jl}$) from amount of unloaded containers ($\sum_{k=1}^{n} Z_{kj}$), it will be possible to determine balance of containers in the port *j*. The excess of laden containers each time turns into the same amount of empty containers, subject to transportation from port:

$$\sum_{k=1}^{n} Z_{kj} \cdot y_k - \sum_{l=1}^{n} Z_{jl} \cdot y_l = \sum_{l=1}^{n} z_{jl} \cdot y_l - \sum_{k=1}^{n} z_{kj} \cdot y_k \,, \ \forall j: \ y_j = 1. \qquad (13)$$

or

$$\left(\sum_{k=1}^{n} Z_{kj} \cdot y_k + \sum_{k=1}^{n} z_{kj} \cdot y_k - \sum_{l=1}^{n} Z_{jl} \cdot y_l - \sum_{l=1}^{n} z_{jl} \cdot y_l \right) \cdot y_j = 0, \ \ j = \overline{1, n}$$

Limitation (13) is the condition of number balance in each port, i.e. empty containers do not appear in the system and outside of ports, but they are the result of containers transportation with cargo.

The efficiency of above stated model was verified at [6]. The model had to find an optimal circular route between 6 ports: Shanghai (China), Ningde (China), Ningbo (China), Napoli (Italy), Odessa (Ukraine) and Novorossiysk (Russian Federation). Port of Napoli was added to the list of ports intentionally. Due to its distance from the main route the model excluded Napoli port from the final route. The final optimal route is: Shanghai (China) - Ningbo (China) - Ningde (China) - Odessa (Ukraine) - Novorossiysk (Russian Federation) - Shanghai (China). This circular route will take 76,7 days. As a rule, Black Sea ports import laden containers from China that in several days turn into empty containers (that have to be delivered back to China ports). Taking into consideration this fact, the optimal distribution of returned empty containers in China ports was also found.

4 CONCLUSIONS

Thus, the suggested model generalizes the TSP problem and permits to optimize a route of container line taking into account container specifics and, unlike mentioned in the article similar models, has a number of advantages:

1. It takes into account variables of route, variables of cargo and variables of time separately. Simultaneous use of these variables at route optimization permits to account, except expenses for transfers, also profits from these transfers and their duration.

2. It optimizes routes according to criterion of maximal profit intensity, i.e. gaining profit in a time unit. This criterion is especially urgent for vessel operators under market conditions, because high profit intensity permits to create short-term credit lines for various projects with short payback period.

3. It permits to exclude from the number of suggested ports prospectless ones (with little cargo flow, with high cost of call or loading-unloading services, etc.).

4. It accepts container flow at each port as not fixed, but depending on whole route time, i.e. the longer the route is, the more containers with cargo will be accumulated in the examined port.

5. It considers cost of calling at port and cost of loading-unloading operations. This permits to select ports not only depending on quantity of containers offered for transportation, but also with consideration of above mentioned expenses.

6. It differs transportation of laden and empty containers. The balance of laden and empty containers in the proposed system is ensured.

Thus, the proposed model significantly extends and generalizes the classical traveling salesman problem. The application of maritime transport specifics allowed to create a new model, which is intended exclusively for finding circular route between sea ports. This model takes into account not only theoretical but also practical aspects of maritime shipping that allows to apply it in practice while creating new container routes.

The main advantage of the model is the optimization criterion - maximal profit intensity. This criterion is crucial for practical applications, as it allows private companies to optimize their profits not only within a particular route (in this case, the company may incur losses on some directions or routes, which then may be covered by the profits earned on other directions), but also for every particular day. This criterion allows companies to find a more optimal route and thus - to gain higher profits.

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