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APPROXIMATION OF A CATENARY FORMED OUT OF A ROPE ACCORDING TO THE HEIGHTS OF ITS POINTS OF HOLD

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Abstract: *When involved in field measurements, we frequently employ a kind of rope or measuring tape fastened to two different points and stretched out across a distance. In order for measurements using this tool to be more accurate, it is necessary to acknowledge and examine the sagging action of a rope or measuring tape when used in this way, as instead of acting as a straight line, it assumes the shape of a kind of curve, a catenary, the parameters of which are influenced by many factors, one of which is always the height difference or lack thereof between the points where the rope is held. In this paper, we assume that most ropes used for this purpose are of a uniform weight distribution, undamaged, and of no elasticity, as well as used in an ideal environment of uniform external impact upon measurements, and aim to examine the effect of different heights of points of hold of a rope upon the catenary its curve forms, as well as attempt to define and anylse this catenary by approximating it into a polynomial.*

Key words: *catenary, parameters, adjustment, polynomial approximation, standard deviation*

1 INTRODUCTION

1.1 An Elementary Definition of a Catenary

When we talk about field observations obtained with the aid of a rope or measuring tape, held in two different points and at a particular height, it is necessary to acknowledge that the rope or measuring tape will sag at points where it is not firmly held, and therefore also assume a particular shape that will not be a straight line, which would invariably be the ideal in such a situation. This shape achieved through the sagging of a rope or measuring tape is referred to as a catenary in mathematics [1] [2], and is influenced primarily by the strain in the points that hold the rope or measuring tape in place, its weight, its weight distribution i.e. density of different segments of the rope, and the distance between the two points where the rope is held firmly i.e. the distance over which the sagging is permitted to happen.

As any of these factors may influence field observations, we have put much effort into analysing them, as well as analysing the catenary formed by the rope in such a way as to minimise its effects on the measurements we obtain through the use of ropes and measuring tapes on the field. It is usually possible to completely discard at least the effect of a different weight distribution within the rope, as that is more of an anomaly than a regular and significant occurrence in ropes and measuring tapes. Thus we have instead focused on analysing how a catenary formed out of a sagging rope is determined by the differences in the points where it is held firmly, preimarily their height.

2 METHOD

2.1 A Method for the Determination of a Catenary

As the rope or measuring tape forms a sagging shape, which is essentially a form of curve, we have approximated the sagging shape of the catenary to the forms of curves we are already familiar with from mathematical theory and found that the catenary corresponds closely with the kind of curve formed by a polynomial consisting of 25 terms, with 25 coefficients. Upon closer inspection, however, we found that there were only two coefficients in this polynomial that, when examined, presented with an adequate impact and degree of accuracy to actually be able to determine the shape of the catenary. We naturally kept only these two coefficients in our approximation and discarded the remaining 23 coefficients, which had an entirely negligible impact on the shape of the catenary, and extensive experimental observations have confirmed that this polynomial with two remaining coefficients matches the shape of a catenary.

2.2 Determination of a Catenary with Points of Hold at an Equal Height

We decided to first examine the most basic sagging shape we would be able to come across in our field observations, which is that of a catenary formed between two points of hold that are at the same height, or rather that have a height difference of zero ($\delta h = 0$). In such a case, the maximum degree of sagging happens at exactly the mid-point of the distance between the points of hold, at s_c

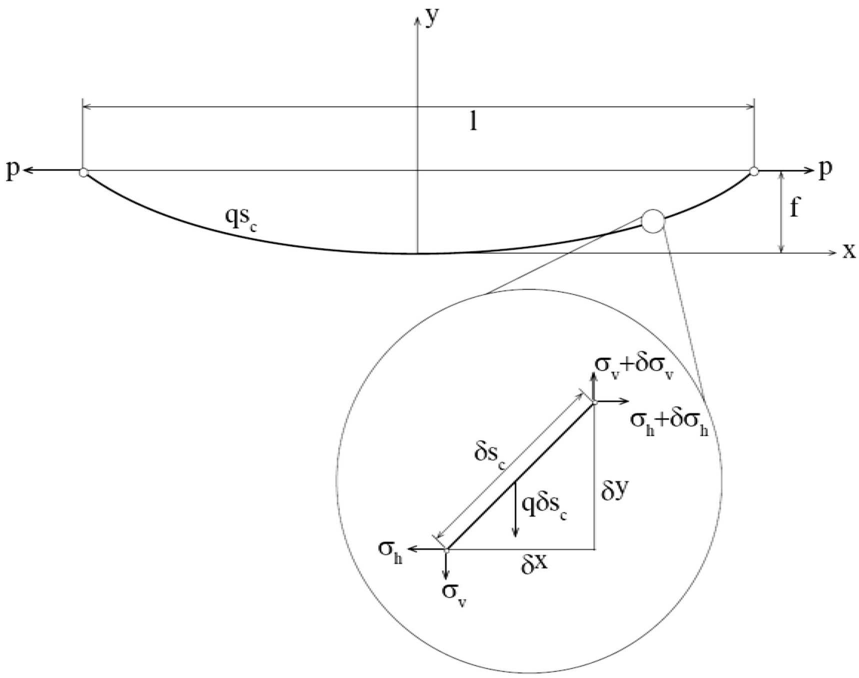

Figure 1 *A catenary formed when points of hold are at equal heights.*

An expression for the appropriate distance correction in such a case is derived with the aid of static balance conditions for ropes burdened only with their own weight. [3] As such, the balance condition in the direction of the x-axis is $\sigma_h + \delta\sigma_h - \sigma_h = 0$ and tells us that the tensile strain in every point of the catenary is constant, and the balance condition in the direction of the y-axis is $\sigma_v + \delta\sigma_v - \sigma_v - q \cdot \delta c_c = 0$ and tells us that the change in tension in the direction of the y-axis is proportional to the specific weight of the rope, i.e. measuring tape, or $\delta\sigma_v = q \cdot \delta s_c$. [4]

An element of the catenary is expressed simply as $\delta s_c = \sqrt{\left(\delta x^2 + \delta y^2\right)} = \delta x \cdot \sqrt{1 + \left(\delta y / \delta x\right)^2}$, and the inclination angle of the catenary also simply as $\tan\varphi = \delta y / \delta x = \sigma_v / \sigma_h$. [5] [6]

Knowing this, if we insert the expression for an element of the catenary, $\delta s_c = \sqrt{\left(\delta x^2 + \delta y^2\right)} = \delta x \cdot \sqrt{1 + \left(\delta y / \delta x\right)^2}$, into the expression detailing the consequences of the balance condition of the y-axis, $\delta\sigma_v = q \cdot \delta s_c$, we get a differential equation of the following form:

$$\delta\sigma_v / \delta x = q \cdot \sqrt{1 + \left(\delta y / \delta x\right)^2} \tag{1}$$

If we then take the derivative of the equation for the inclination angle of the catenary, $\tan\varphi = \delta y / \delta x = \sigma_v / \sigma_h$, and substitute it into the equation for an element of the catenary, $\delta s_c = \sqrt{\left(\delta x^2 + \delta y^2\right)} = \delta x \cdot \sqrt{1 + \left(\delta y / \delta x\right)^2}$, we get another differential equation, this time of the following form:

$$\sigma_h \cdot \delta y^2 / \delta x^2 = q \cdot \sqrt{1 + \left(\delta y / \delta x\right)^2} \tag{2}$$

The solution to this differential equation is then:

$$y = a \cdot ch\frac{x}{a} - K \tag{3}$$

Here, a is the paramater of the catenary, defined through the expression $\frac{\delta_h}{q}$, K is the integration constant, and q is the specific weight of the rope or measuring tape.

If the coordinate origin is placed into the vertex of the catenary, the integration constant is equal to $-a$. Because the greatest degree of sagging of the rope is displayed at $\frac{1}{2}s_c$, the expression for the curve of the catenary can be written down in the following form:

$$f = a \cdot \left(ch\frac{s_c}{2 \cdot a} - 1\right) \tag{4}$$

The hyperbolic cosine [7] can then be developed into a power series with the help of the following expression:

$$chx = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots + \frac{x^{2n}}{(2n)!} + \ldots \tag{5}$$

And if we take note of the first two terms of the series and the parameter of the catenary, we then get an expression for the largest degree of sagging of the rope or measuring tape, which is:

$$f = \frac{q \cdot s_c^2}{8 \cdot \sigma_h} \tag{6}$$

And the length of the catenary is then:

$$s_c = \int ds_c = \int \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx = a \cdot sh\frac{x}{a} \tag{7}$$

The hyperbolic sine [8] can then also be developed into a power series with the help of the following expression:

$$shx = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots + \frac{x^{2n+1}}{(2n+1)!} + \ldots \tag{8}$$

And if we take note of the first two terms of the series, we then get an expression for the length of the catenary, which is:

$$s_c = 2 \cdot a \cdot sh\frac{s_c}{2 \cdot a} \approx 2a \cdot \left(\frac{s_c}{2a} + \frac{s_c^3}{48a^3}\right) = s_c + \frac{s_c^3 \cdot q^2}{24 \cdot \sigma_h^2} = s_c + \frac{8 \cdot f^2}{3 \cdot s_c} \tag{9}$$

The correction due to the sagging of the rope or measuring tape is therefore defined with the aid of the following expression:

$$\delta l = \frac{Q^2 \cdot s_c}{24 \cdot p^2} = \frac{q^2 \cdot s_c^3}{24 \cdot p^2} \tag{10}$$

Here, Q is the weight of the rope or measuring tape, s_c is the observed length of the catenary, and $\sigma_h = p$ is the force of the tension of the rope or measuring tape.

2.3 Determination of a Catenary with Points of Hold at a Different Height

We can now proceed to examine the a more complex sagging shape that we would be able to come across in our field observations, which is that of a catenary formed between two points of hold that are at different heights, or rather that have a height difference that is not equal to zero ($\delta h \neq 0$). In such a case, the maximum degree of sagging does not happen at exactly the mid-point of the distance between the points of hold, and the expression we derived above for a catenary formed between two points of hold that are at an equal height does not hold true. [9]

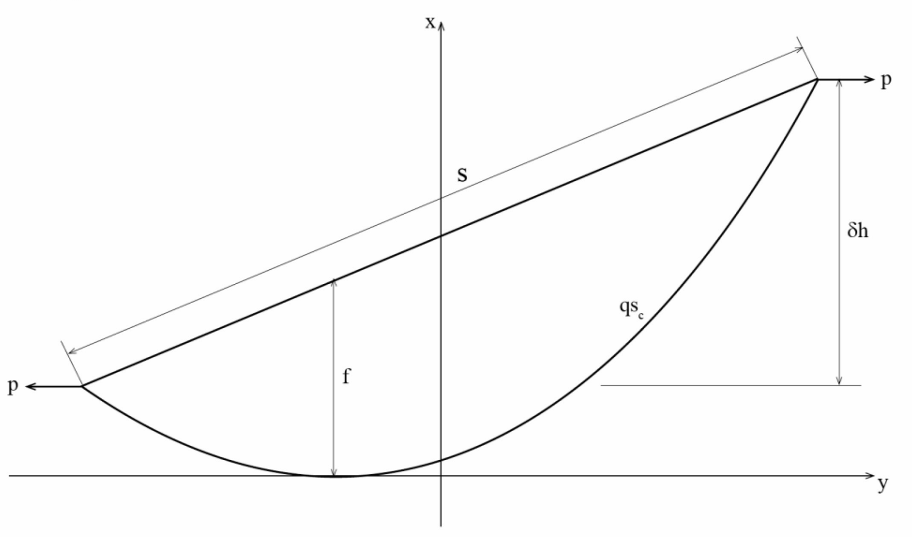

Figure 2 *A catenary formed when points of hold are at different heights*

2.4 The General Mathematical Definition of a Catenary

In order to be able to better examine the above situation, we need to have a complex understanding of the mathematical definition of a catenary. It is a curve with the shape of a hyperbolic cosine and is mathematically defined with the aid of the following expression:

$$y = a \cdot ch\frac{x}{a} = a \cdot \frac{e^{\frac{x}{a}} + e^{-\frac{x}{a}}}{2} \tag{11}$$

When we use this expression, we presume that the rope or measuring tape that takes the shape of this curve is homogenous and unstretchable. In this expression, we employ the parameter a to mark the height difference in the catenary, i.e. the parameter is used to define the vertex of the curve in the point $A(0, a)$. [10] The curve is symmetrical in relation to the y-axis and lies above the curve of the parabola, as is understood with the aid of the following expression:

$$y = a + \frac{x^2}{2 \cdot a} \tag{12}$$

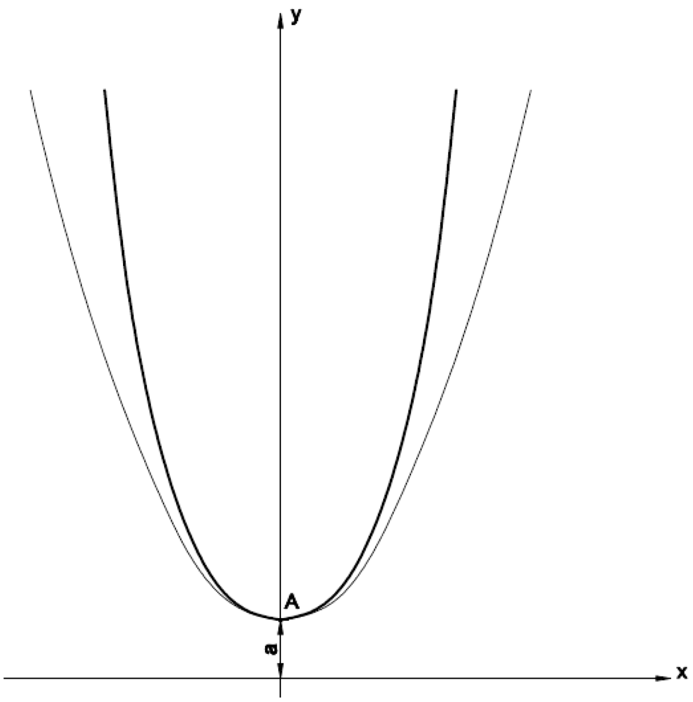

Figure 3 *A depiction of a catenary and a parabola.*

2.5 Our Approach to the General Mathematical Determination of a Catenary

When we are dealing with two points of hold that are at different heights, the catenary formed between them can be determined with the aid of an adjustment of terms appearing in a mathematical series. The factors that influence the shape of the catenary most are the observed distance of the catenary and the relative height difference of the points of hold of the catenary, and the latter is easily taken from RTK-GNSS observations, the accuracy of which is absolute in comparison to the accuracy of the measurements and can thus be neglected.

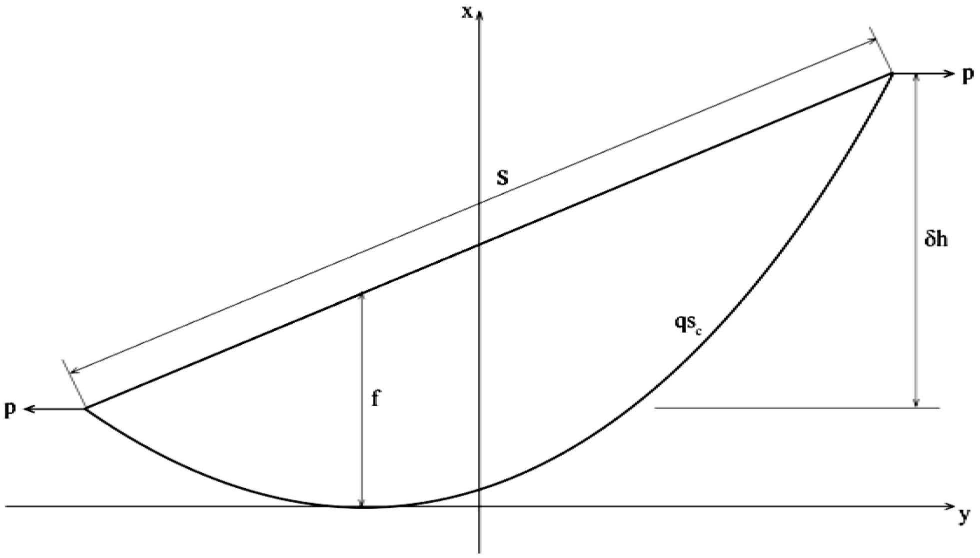

Figure 4 *A catenary formed when points of hold are at different heights.*

A catenary is defined as a hyperbolic cosine, the shape of which can be approximated with its development into a convergent series. [11] That is how the beginning of all 25 terms of the series makes up an expression describing the shape of its chord, i.e. its diagonal distance.

$$s = \begin{vmatrix} a_{2,2}s_c^2h^2 & a_{2,1}s_c^2h & a_{2,0}s_c^2 & a_{2,-1}s_c^2\dfrac{1}{h} & a_{2,-2}s_c^2\dfrac{1}{h^2} \\ a_{1,2}s_ch^2 & a_{1,1}s_ch & a_{1,0}s_c & a_{1,-1}s_c\dfrac{1}{h} & a_{1,-2}s_c\dfrac{1}{h^2} \\ a_{0,2}h^2 & a_{0,1}h & a_{0,0} & a_{0,-1}\dfrac{1}{h} & a_{0,-2}\dfrac{1}{h^2} \\ a_{-1,2}\dfrac{1}{s_c}h^2 & a_{-1,1}\dfrac{1}{s_c}h & a_{-1,0}\dfrac{1}{s_c} & a_{-1,-1}\dfrac{1}{s_ch} & a_{-1,-2}\dfrac{1}{s_ch^2} \\ a_{-2,2}\dfrac{1}{s_c^2}h^2 & a_{-2,1}\dfrac{1}{s_c^2}h & a_{-2,0}\dfrac{1}{s_c^2} & a_{-2,-1}\dfrac{1}{s_c^2h} & a_{-2,-2}\dfrac{1}{s_c^2h^2} \end{vmatrix} \tag{13}$$

Here, a is the coefficient of a particular term of the series, with indexes i and j that determine the sequential term of the series, s_c is the length of the arc of the catenary, and h is a computer-determined relative height difference, gained from RTK-GNSS observational data.

In *Table 2*, the martix of coefficients, $\underset{51\text{x}25}{A}$, is displayed. Each measured value has a belonging equation of corrections, made up from terms of the series.

$$\underset{51x25}{\mathbf{A}} = \begin{vmatrix} \left(s_c^2 h^2\right)_1 & \left(s_c^2 h\right)_2 & \left(s_c^2\right)_3 & . & . & . & \left(s_c^i h^j\right)_u \\ \left(s_c^2 h^2\right)_2 & . & & & & & . \\ \left(s_c^2 h^2\right)_3 & & . & & & & . \\ . & & & . & & & . \\ . & & & & . & & . \\ . & & & & & . & . \\ \left(s_c^2 h^2\right)_n & . & . & . & . & . & \left(s_c^i h^j\right)_{nxu} \end{vmatrix} \tag{14}$$

3 RESULTS

Here, the basis for the determination of the vector of free terms is the diagonal length [12] determined from the RTK-GNSS observations.

Tab. 1 *A table depicting the contents of the martix of coefficients.*

ll*hh	ll*h	ll*0	ll*1/h	ll*1/hh	l*hh	l*h	l*0	l*1/h	l*1/hh	0*hh	0*h	0*0	0*1/h	0*1/hh	1/l*hh	1/l*h	1/l*0	1/l*1/h	1/l*1/hh	1.
0,38	3,26	28,09	241,95	2083,95	0,07	0,62	5,30	45,65	393,20	0,0135	0,12	1	8,61	74,19	0,0025	0,0219	0,1887	1,6251	13,9978	
0,22	-2,45	27,04	-298,78	3301,49	0,04	-0,47	5,20	-57,46	634,90	0,0082	-0,09	1	-11,05	122,10	0,0016	-0,0174	0,1923	-2,1249	23,4801	
350,18	-310,64	275,56	-244,44	216,84	21,10	-18,71	16,60	-14,73	13,06	1,2708	-1,13	1	-0,89	0,79	0,0766	-0,0679	0,0602	-0,0534	0,0474	
16,04	48,07	144,00	431,40	1292,38	1,34	4,01	12,00	35,95	107,70	0,1114	0,33	1	3,00	8,97	0,0093	0,0278	0,0833	0,2497	0,7479	
12,12	33,42	92,16	254,16	700,95	1,26	3,48	9,60	26,48	73,02	0,1315	0,36	1	2,76	7,61	0,0137	0,0378	0,1042	0,2873	0,7923	
14,71	39,12	104,04	276,70	735,91	1,44	3,84	10,20	27,13	72,15	0,1414	0,38	1	2,66	7,07	0,0139	0,0369	0,0980	0,2607	0,6935	
11,61	38,16	125,44	412,36	1355,56	1,04	3,41	11,20	36,82	121,03	0,0925	0,30	1	3,29	10,81	0,0083	0,0272	0,0893	0,2935	0,9649	
388,82	216,90	121,00	67,50	37,65	35,35	19,72	11,00	6,14	3,42	3,2134	1,79	1	0,56	0,31	0,2921	0,1630	0,0909	0,0507	0,0283	
80,02	82,30	84,64	87,05	89,53	8,70	8,95	9,20	9,46	9,73	0,9454	0,97	1	1,03	1,06	0,1028	0,1057	0,1087	0,1118	0,1150	
14,58	24,05	39,69	65,50	108,08	2,31	3,82	6,30	10,40	17,16	0,3672	0,61	1	1,65	2,72	0,0583	0,0962	0,1587	0,2619	0,4322	
536,01	449,15	376,36	315,37	264,26	27,63	23,15	19,40	16,26	13,62	1,4242	1,19	1	0,84	0,70	0,0734	0,0615	0,0515	0,0432	0,0362	
795,13	-648,55	529,00	-431,48	351,94	34,57	-28,20	23,00	-18,76	15,30	1,5031	-1,23	1	-0,82	0,67	0,0654	-0,0533	0,0435	-0,0355	0,0289	
121,93	-164,53	222,01	-299,57	404,22	8,18	-11,04	14,90	-20,11	27,13	0,5492	-0,74	1	-1,35	1,82	0,0369	-0,0497	0,0671	-0,0906	0,1222	
154,87	-129,42	108,16	-90,39	75,54	14,89	-12,44	10,40	-8,69	7,26	1,4319	-1,20	1	-0,84	0,70	0,1377	-0,1151	0,0962	-0,0804	0,0672	
2,86	23,35	190,44	1553,34	12670,02	0,21	1,69	13,80	112,56	918,12	0,0150	0,12	1	8,16	66,53	0,0011	0,0089	0,0725	0,5911	4,8210	
776,12	325,95	136,89	57,49	24,14	66,33	27,86	11,70	4,91	2,06	5,6696	2,38	1	0,42	0,18	0,4846	0,2035	0,0855	0,0359	0,0151	
43,90	-80,17	146,41	-267,37	488,25	3,63	-6,63	12,10	-22,10	40,35	0,2999	-0,55	1	-1,83	3,33	0,0248	-0,0453	0,0826	-0,1509	0,2756	
1,69	-7,55	33,64	-149,98	668,65	0,29	-1,30	5,80	-25,86	115,28	0,0503	-0,22	1	-4,46	19,88	0,0087	-0,0387	0,1724	-0,7687	3,4270	
1,64	-11,39	79,21	-550,83	3830,56	0,18	-1,28	8,90	-61,89	430,40	0,0207	-0,14	1	-6,95	48,36	0,0023	-0,0162	0,1124	-0,7814	5,4337	
328,58	197,58	118,81	71,44	42,96	30,14	18,13	10,90	6,55	3,94	2,7656	1,66	1	0,60	0,36	0,2537	0,1526	0,0917	0,0552	0,0332	
36,63	33,29	30,25	27,49	24,98	6,66	6,05	5,50	5,00	4,54	1,2109	1,10	1	0,91	0,83	0,2202	0,2001	0,1818	0,1652	0,1502	
1,31	-17,99	246,49	-3376,58	46254,46	0,08	-1,15	15,70	-215,07	2946,14	0,0053	-0,07	1	-13,70	187,65	0,0003	-0,0046	0,0637	-0,8725	11,9524	
4,52	-21,89	106,09	-514,25	2492,73	0,44	-2,12	10,30	-49,93	242,01	0,0426	-0,21	1	-4,85	23,50	0,0041	-0,0200	0,0971	-0,4706	2,2812	
17,99	29,69	49,00	80,86	133,43	2,57	4,24	7,00	11,55	19,06	0,3672	0,61	1	1,65	2,72	0,0525	0,0866	0,1429	0,2357	0,3890	
100,53	-93,24	86,49	-80,22	74,41	10,81	-10,03	9,30	-8,63	8,00	1,1623	-1,08	1	-0,93	0,86	0,1250	-0,1159	0,1075	-0,0997	0,0925	
115,30	188,98	309,76	507,72	832,19	6,55	10,74	17,60	28,85	47,28	0,3722	0,61	1	1,64	2,69	0,0211	0,0347	0,0568	0,0931	0,1526	
107,69	-147,36	201,64	-275,92	377,55	7,58	-10,38	14,20	-19,43	26,59	0,5341	-0,73	1	-1,37	1,87	0,0376	-0,0515	0,0704	-0,0964	0,1319	
9,71	20,88	44,89	96,50	207,43	1,45	3,12	6,70	14,40	30,96	0,2164	0,47	1	2,15	4,62	0,0323	0,0694	0,1493	0,3208	0,6897	
4,64	13,78	40,96	121,76	361,95	0,72	2,15	6,40	19,02	56,55	0,1132	0,34	1	2,97	8,84	0,0177	0,0526	0,1563	0,4645	1,3807	
1307,29	506,19	196,00	75,89	29,39	93,38	36,16	14,00	5,42	2,10	6,6698	2,58	1	0,39	0,15	0,4764	0,1845	0,0714	0,0277	0,0107	
2526,50	2151,31	1831,84	1559,81	1328,18	59,03	50,26	42,80	36,44	31,03	1,3792	1,17	1	0,85	0,73	0,0322	0,0274	0,0234	0,0199	0,0169	
0,08	3,64	174,24	8336,84	398891,97	0,01	0,28	13,20	631,58	30219,09	0,0004	0,02	1	47,85	2289,32	0,00003	0,0016	0,0758	3,6248	173,4337	0,

The observations are used to obtain the following expression:

$$s_{GPS} = \sqrt{\Delta Y^2 + \Delta X^2 + \Delta H^2} \tag{15}$$

The vector of the free terms, $\underset{18\times 1}{f}$, is then:

$$\underset{51x1}{\mathrm{f}} = \left(\underset{51x25}{\mathrm{A}} \cdot \underset{51x1}{\mathrm{s_{GPS}}} \right) - \underset{1x25}{\mathrm{a}_{ij}^{T}} \tag{16}$$

Tab. 2 *A table depicting the determination of the vector of free terms* $\underset{51\times1}{f}$

CONCATENATE =MMULT(CU8:DT8;TRANSPOSE(CU$5:DT$5))

	CS	CT	CU	CV	CW	CX	CY	CZ	DA	DB	DC	DD	DJ	DK	DL	DM	DN	DO	DP	DQ	DR	DS	DT	DU	EW	EX	FL	FM
4			ll*hh	ll*h	ll*0	ll*1/h	ll*1/hh	l*hh	l*h	l*0	l*1/h	l*1/hh	1/l*hh	1/l*h	1/l*0	1/l*1/h	1/l*1/hh	1/ll*hh	1/ll*h	1/ll*0	1/ll*1/h	1/ll*1/hh	f		f			
5			0,0001	0	0	0	0	0	0	0,99	0	0	0	0	0	0	0	0	0	0	0	0	-1		-1			
6			1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0						
7																												
8	0		0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,000	0,000	0,00	0,00	0,00	0,0000	0,0000	0,0000	0,0000	0,0000	0,00		=MMULT(CU8:DT8;TRANSPOSE(CU$5:DT$5))			
9	0		0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,000	0,000	0,00	0,00	0,00	0,0000	0,0000	0,0000	0,0000	0,0000	0,00					
10	1		350,18	0,00	0,00	0,00	0,00	0,00	0,00	16,60	0,00	0,00	0,000	0,000	0,00	0,00	0,00	0,0000	0,0000	0,0000	0,0000	0,0000	16,43		-0,01			
13	1		14,71	0,00	0,00	0,00	0,00	0,00	0,00	10,20	0,00	0,00	0,000	0,000	0,00	0,00	0,00	0,0000	0,0000	0,0000	0,0000	0,0000	10,26		-0,19			
14	1		11,61	0,00	0,00	0,00	0,00	0,00	0,00	11,20	0,00	0,00	0,000	0,000	0,00	0,00	0,00	0,0000	0,0000	0,0000	0,0000	0,0000	11,05		0,00			
15	1		388,82	0,00	0,00	0,00	0,00	0,00	0,00	11,00	0,00	0,00	0,000	0,000	0,00	0,00	0,00	0,0000	0,0000	0,0000	0,0000	0,0000	10,99		-0,09			
16	0		0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,000	0,000	0,00	0,00	0,00	0,0000	0,0000	0,0000	0,0000	0,0000	0,00		0,00			
17	1		671,34	0,00	0,00	0,00	0,00	0,00	0,00	18,10	0,00	0,00	0,000	0,000	0,00	0,00	0,00	0,0000	0,0000	0,0000	0,0000	0,0000	17,95		-0,01			
18	0		0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,000	0,000	0,00	0,00	0,00	0,0000	0,0000	0,0000	0,0000	0,0000	0,00		0,00			
19	0		0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,000	0,000	0,00	0,00	0,00	0,0000	0,0000	0,0000	0,0000	0,0000	0,00		0,00			
20	1		28,61	0,00	0,00	0,00	0,00	0,00	0,00	9,70	0,00	0,00	0,000	0,000	0,00	0,00	0,00	0,0000	0,0000	0,0000	0,0000	0,0000	9,52		0,05			
28	1		32,48	0,00	0,00	0,00	0,00	0,00	0,00	12,00	0,00	0,00	0,000	0,000	0,00	0,00	0,00	0,0000	0,0000	0,0000	0,0000	0,0000	11,94		-0,10			
29	0		0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,000	0,000	0,00	0,00	0,00	0,0000	0,0000	0,0000	0,0000	0,0000	0,00		0,00			
34	1		22,91	0,00	0,00	0,00	0,00	0,00	0,00	9,80	0,00	0,00	0,000	0,000	0,00	0,00	0,00	0,0000	0,0000	0,0000	0,0000	0,0000	9,60		0,06			
35	0		0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,000	0,000	0,00	0,00	0,00	0,0000	0,0000	0,0000	0,0000	0,0000	0,00		0,00			
36	0		0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,000	0,000	0,00	0,00	0,00	0,0000	0,0000	0,0000	0,0000	0,0000	0,00		0,00			
37	1		536,01	0,00	0,00	0,00	0,00	0,00	0,00	19,40	0,00	0,00	0,000	0,000	0,00	0,00	0,00	0,0000	0,0000	0,0000	0,0000	0,0000	19,16		0,04			
38	1		796,13	0,00	0,00	0,00	0,00	0,00	0,00	23,00	0,00	0,00	0,000	0,000	0,00	0,00	0,00	0,0000	0,0000	0,0000	0,0000	0,0000	22,73		0,06			
39	1		121,93	0,00	0,00	0,00	0,00	0,00	0,00	14,90	0,00	0,00	0,000	0,000	0,00	0,00	0,00	0,0000	0,0000	0,0000	0,0000	0,0000	14,52		0,19			
40	1		154,87	0,00	0,00	0,00	0,00	0,00	0,00	10,40	0,00	0,00	0,000	0,000	0,00	0,00	0,00	0,0000	0,0000	0,0000	0,0000	0,0000	10,29		-0,009			
46	1		328,58	0,00	0,00	0,00	0,00	0,00	0,00	10,90	0,00	0,00	0,000	0,000	0,00	0,00	0,00	0,0000	0,0000	0,0000	0,0000	0,0000	10,77		0,02			
47	0		0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,000	0,000	0,00	0,00	0,00	0,0000	0,0000	0,0000	0,0000	0,0000	0,00		0,00			
48	1		1,31	0,00	0,00	0,00	0,00	0,00	0,00	15,70	0,00	0,00	0,000	0,000	0,00	0,00	0,00	0,0000	0,0000	0,0000	0,0000	0,0000	15,48		0,01			
49	1		4,52	0,00	0,00	0,00	0,00	0,00	0,00	10,30	0,00	0,00	0,000	0,000	0,00	0,00	0,00	0,0000	0,0000	0,0000	0,0000	0,0000	10,18		-0,02			
50	0		0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,000	0,000	0,00	0,00	0,00	0,0000	0,0000	0,0000	0,0000	0,0000	0,00		0,00			
51	0		0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,000	0,000	0,00	0,00	0,00	0,0000	0,0000	0,0000	0,0000	0,0000	0,00		0,00			
52	1		115,30	0,00	0,00	0,00	0,00	0,00	0,00	17,60	0,00	0,00	0,000	0,000	0,00	0,00	0,00	0,0000	0,0000	0,0000	0,0000	0,0000	17,28		0,09			
53	1		107,69	0,00	0,00	0,00	0,00	0,00	0,00	14,20	0,00	0,00	0,000	0,000	0,00	0,00	0,00	0,0000	0,0000	0,0000	0,0000	0,0000	14,01		0,01			

In *Table 3*, the switches for turning unknowns and measurements on and off is taken into account, and the switches are denoted with a yellow hue. The coefficients for the terms of the series are also expressed, and denoted with a blue hue. The diagonal lengths gained from the previously-mentioned RTK-GNSS observations, determined with the aid of the expression $s_{GPS} = \sqrt{\Delta Y^2 + \Delta X^2 + \Delta H^2}$, while also taking switches into account, are expressed in the form of values beneath the cell $\boldsymbol{f}$.

In continuation, when the matrix of coefficients, $\underset{51x25}{\mathbf{A}}$, and the vector of free terms, $\underset{51\times1}{\mathbf{f}}$, are both determined, the matrix of coefficients of normal equations, $\underset{25x25}{\mathbf{N}}$, and the vector of free terms of normal equations, $\underset{25x25}{n}$, are also both determined [15] thusly:

$$\underset{25x25}{N} = \underset{25x51}{A^T} \cdot \underset{51x25}{A} \tag{17}$$

$$\underset{25x1}{n} = \underset{25x51}{A^T} \cdot \underset{51x1}{f} \tag{18}$$

Tab. 3 *A matrix of the coefficients of* $\underset{25x25}{N}$ *and the vector of the free terms of normal equations*

SUM =MMULT(TRANSPOSE(DX8:EW58);DX8:EW58)

	DW	DX	DY	DZ	EA	EB	EM	EN	EO	EP	EQ	ER	ES	ET	EU	EV	EW
4		ll*hh	ll*h	ll*0	ll*1/h	ll*1/hh	1/l*hh	1/l*h	1/l*0	1/l*1/h	1/l*1/hh	1/ll*hh	1/ll*h	1/ll*0	1/ll*1/h	1/ll*1/hh	f
8		0,000	0,000	0,000	0,000	0,000	0,0000	0,000	0,000	0,000	0,000	0,0000	0,0000	0,0000	0,0000	0,0000	-0,11
48		0,000	0,000	0,000	0,000	0,000	0,0000	0,000	0,000	0,000	0,000	0,0000	0,0000	0,0000	0,0000	0,0000	0,06
49		0,000	0,000	0,000	0,000	0,000	0,0000	0,000	0,000	0,000	0,000	0,0000	0,0000	0,0000	0,0000	0,0000	0,01
50		0,000	0,000	0,000	0,000	0,000	0,0000	0,000	0,000	0,000	0,000	0,0000	0,0000	0,0000	0,0000	0,0000	0,04
51		0,000	0,000	0,000	0,000	0,000	0,0000	0,000	0,000	0,000	0,000	0,0000	0,0000	0,0000	0,0000	0,0000	0,07
52		0,000	0,000	0,000	0,000	0,000	0,0000	0,000	0,000	0,000	0,000	0,0000	0,0000	0,0000	0,0000	0,0000	0,13
53		0,000	0,000	0,000	0,000	0,000	0,0000	0,000	0,000	0,000	0,000	0,0000	0,0000	0,0000	0,0000	0,0000	0,04
54		0,000	0,000	0,000	0,000	0,000	0,0000	0,000	0,000	0,000	0,000	0,0000	0,0000	0,0000	0,0000	0,0000	0,03
55		0,000	0,000	0,000	0,000	0,000	0,0000	0,000	0,000	0,000	0,000	0,0000	0,0000	0,0000	0,0000	0,0000	0,00
56		0,000	0,000	0,000	0,000	0,000	0,0000	0,000	0,000	0,000	0,000	0,0000	0,0000	0,0000	0,0000	0,0000	-0,13
57		0,000	0,000	0,000	0,000	0,000	0,0000	0,000	0,000	0,000	0,000	0,0000	0,0000	0,0000	0,0000	0,0000	-0,23
58		0,000	0,000	0,000	0,000	0,000	0,00000	0,000	0,000	0,000	0,000	0,0000	0,0000	0,0000	0,0000	0,0000	-0,10
59																	
63																	
64	N	ll*hh	ll*h	ll*0	ll*1/h	ll*1/hh	1/l*hh	1/l*h	1/l*0	1/l*1/h	1/l*1/hh	1/ll*hh	1/ll*h	1/ll*0	1/ll*1/h	1/ll*1/hh	n
65	ll*hh	=MMULT(TRANSPOSE(DX8:EW58);DX8:EW58)				0	0	0	0	0	0	0	0	0	0	0	0,0000
66	ll*h	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
67	ll*0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
68	ll*1/h	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
69	ll*1/hh	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
70	l*hh	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
71	l*h	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-31,78913452
72	l*0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	11,53693725
73	l*1/h	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
74	l*1/hh	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
75	0*hh	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
83	1/l*1/h	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
84	1/l*1/hh	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
85	1/ll*hh	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
86	1/ll*h	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
87	1/ll*0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
88	1/ll*1/h	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
89	1/ll*1/hh	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

This adjustment by parametric variation [13] is continued with the addition of an auxilliary matrix, $\underset{25x25}{N\Delta}$, with the aid of which the process of adjustment is automatised, as the determinant of the matrix of coefficients of normal equations, $\underset{25x25}{N}$, is equal to zero, and that is why it is not possible to directly invert it and determine the matrix of cofactors of the sought-after values $\underset{25x25}{Q_{xx}} = \underset{25x25}{N^{-1}}$.

This auxilliary matrix, $\underset{25x25}{N\Delta}$, is formed out of the matrix of coefficients, $\underset{51x25}{A}$, and thus, when its terms are positive, null or negative, an auxilliary vector with values of $+1$, 0 or -1, respectively, is formed. This matrix is then gained with the aid of the expression written down in the following table:

Tab. 4 *The auxilliary matrix* $\underset{25x25}{N}\Delta$.

SUM =($DW95=DX$94)*(1-DX$92)

	DV	DW	DX	DY	DZ	EA	EB	EC	ED	EE	EF	EG	EH	EI
55	1		0,000	0,000	0,000	0,000	0,000	0,000	2,153	6,400	0,000	0,000	0,000	0,000
56	1		0,000	0,000	0,000	0,000	0,000	0,000	36,156	14,000	0,000	0,000	0,000	0,000
57	1		0,000	0,000	0,000	0,000	0,000	0,000	50,264	42,800	0,000	0,000	0,000	0,000
58	1		0,000	0,000	0,000	0,000	0,000	0,000	0,276	13,200	0,000	0,000	0,000	0,000
59														
60			0,00	0,00	0,00	0,00	0,00	0,00	9018,81	7926,47	0,00	0,00	0,00	0,00
92			0	0	0	0	0	0	1	1	0	0	0	0
93														
94		NΔ	II*hh	II*h	II*0	II*1/h	II*1/hh	I*hh	I*h	I*0	I*1/h	I*1/hh	0*hh	0*h
95		II*hh	=($DW95=DX$94)*(1-DX$92)		0	0	0	0	0	0	0	0	0	0
96		II*h	0	1	0	0	0	0	0	0	0	0	0	0
97		II*0	0	0	1	0	0	0	0	0	0	0	0	0
98		II*1/h	0	0	0	1	0	0	0	0	0	0	0	0
99		II*1/hh	0	0	0	0	1	0	0	0	0	0	0	0
100		I*hh	0	0	0	0	0	1	0	0	0	0	0	0
101		I*h	0	0	0	0	0	0	0	0	0	0	0	0
102		I*0	0	0	0	0	0	0	0	0	0	0	0	0
103		I*1/h	0	0	0	0	0	0	0	0	1	0	0	0
104		I*1/hh	0	0	0	0	0	0	0	0	0	1	0	0
105		0*hh	0	0	0	0	0	0	0	0	0	0	1	0
106		0*h	0	0	0	0	0	0	0	0	0	0	0	1
107		0*0	0	0	0	0	0	0	0	0	0	0	0	0
108		0*1/h	0	0	0	0	0	0	0	0	0	0	0	0
109		0*1/hh	0	0	0	0	0	0	0	0	0	0	0	0
110		1/I*hh	0	0	0	0	0	0	0	0	0	0	0	0
111		1/I*h	0	0	0	0	0	0	0	0	0	0	0	0
112		1/I*0	0	0	0	0	0	0	0	0	0	0	0	0

The auxilliary matrix, $\underset{25x25}{N\Delta}$, is then counted into matrix $\underset{25x25}{N}$, thereby gaining matrix $\underset{25x25}{N_{full}}$, which is then inverted. The result of this inversion is then a matrix of cofactors of the sought-after values of $\underset{25x25}{Q_{xx}}$. The vector of growth of the sought-after values $\underset{25x1}{x}$ is then also determined, and thus:

$$\underset{25x1}{x} = \underset{25x25}{Q_{xx}} \cdot \underset{25x1}{n} \tag{19}$$

Tab. 5 *A matrix of the cofactors of* $\underset{25\times25}{Q_{xx}}$.

SUM =MINVERSE(DX123:EV147)

	DW	DX	DY	DZ	EA	EB	EC	ED	EE	EP	EQ	ER	ES	ET	EU	EV
122	N_{full}	ll*hh	ll*h	ll*0	ll*1/h	ll*1/hh	l*hh	l*h	l*0	1/l*1/h	1/l*1/hh	1/ll*hh	1/ll*h	1/ll*0	1/ll*1/h	1/ll*1/hh
123	ll*hh	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
124	ll*h	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
125	ll*0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
126	ll*1/h	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
127	ll*1/hh	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
128	l*hh	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
129	l*h	0	0	0	0	0	0	9018,812949	2668,919914	0	0	0	0	0	0	0
130	l*0	0	0	0	0	0	0	2668,919914	7926,465	0	0	0	0	0	0	0
131	l*1/h	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
132	l*1/hh	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
144	1/ll*h	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
145	1/ll*0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
146	1/ll*1/h	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
147	1/ll*1/hh	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
150																
151	Q_{xx}	ll*hh	ll*h	ll*0	ll*1/h	ll*1/hh	l*hh	l*h	l*0	1/l*1/h	1/l*1/hh	1/ll*hh	1/ll*h	1/ll*0	1/ll*1/h	1/ll*1/hh
152	ll*hh	=MINVERSE(DX123:EV147)		0	0	0	0	0	0	0	0	0	0	0	0	0
153	ll*h	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
154	ll*0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
155	ll*1/h	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
156	ll*1/hh	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
157	l*hh	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
158	l*h	0	0	0	0	0	0	0,00012315	-4,14659E-05	0	0	0	0	0	0	0
159	l*0	0	0	0	0	0	0	-4,14659E-05	0,000140122	0	0	0	0	0	0	0
160	l*1/h	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
161	l*1/hh	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
169	1/l*0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
170	1/l*1/h	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
171	1/l*1/hh	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
172	1/ll*hh	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
173	1/ll*h	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
174	1/ll*0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
175	1/ll*1/h	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
176	1/ll*1/hh	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

Tab. 6 *The vector of growth of the sought-after values of* $\underset{25\times1}{x}$.

SUM =MMULT(DX152:EV176;EW65:EW89)

	DW	DX	EC	ED	EE	EP	EQ	ER	ES	ET	EU	EV	EW	EX
64	N	ll*hh	l*hh	l*h	l*0	1/l*1/h	1/l*1/hh	1/ll*hh	1/ll*h	1/ll*0	1/ll*1/h	1/ll*1/hh	n	
65	ll*hh	0	0	0	0	0	0	0	0	0	0	0	0,0000	
66	ll*h	0	0	0	0	0	0	0	0	0	0	0	0	
67	ll*0	0	0	0	0	0	0	0	0	0	0	0	0	
68	ll*1/h	0	0	0	0	0	0	0	0	0	0	0	0	
69	ll*1/hh	0	0	0	0	0	0	0	0	0	0	0	0	
70	l*hh	0	0	0	0	0	0	0	0	0	0	0	0	
71	l*h	0	0	9018,812949	2668,919914	0	0	0	0	0	0	0	-31,78913452	
72	l*0	0	0	2668,919914	7926,465	0	0	0	0	0	0	0	11,53693725	
73	l*1/h	0	0	0	0	0	0	0	0	0	0	0	0	
74	l*1/hh	0	0	0	0	0	0	0	0	0	0	0	0	
85	1/ll*hh	0	0	0	0	0	0	0	0	0	0	0	0	
86	1/ll*h	0	0	0	0	0	0	0	0	0	0	0	0	
87	1/ll*0	0	0	0	0	0	0	0	0	0	0	0	0	
88	1/ll*1/h	0	0	0	0	0	0	0	0	0	0	0	0	
89	1/ll*1/hh	0	0	0	0	0	0	0	0	0	0	0	0	
150														
151	Q_{xx}	ll*hh	l*hh	l*h	l*0	1/l*1/h	1/l*1/hh	1/ll*hh	1/ll*h	1/ll*0	1/ll*1/h	1/ll*1/hh	x	
152	ll*hh	1	0	0	0	0	0	0	0	0	0	0	=MMULT(DX152:EV176;EW65:EW89)	
153	ll*h	0	0	0	0	0	0	0	0	0	0	0	0	
154	ll*0	0	0	0	0	0	0	0	0	0	0	0	0	
155	ll*1/h	0	0	0	0	0	0	0	0	0	0	0	0	
156	ll*1/hh	0	0	0	0	0	0	0	0	0	0	0	0	
157	l*hh	0	1	0	0	0	0	0	0	0	0	0	0	
158	l*h	0	0	0,00012315	-4,14659E-05	0	0	0	0	0	0	0	-0,00439323	
159	l*0	0	0	-4,14659E-05	0,000140122	0	0	0	0	0	0	0	0,00293474	
160	l*1/h	0	0	0	0	0	0	0	0	0	0	0	0	
161	l*1/hh	0	0	0	0	0	0	0	0	0	0	0	0	
169	1/l*0	0	0	0	0	0	0	0	0	0	0	0	0	
170	1/l*1/h	0	0	0	0	1	0	0	0	0	0	0	0	
171	1/l*1/hh	0	0	0	0	0	1	0	0	0	0	0	0	
172	1/ll*hh	0	0	0	0	0	0	1	0	0	0	0	0	
173	1/ll*h	0	0	0	0	0	0	0	1	0	0	0	0	
174	1/ll*0	0	0	0	0	0	0	0	0	1	0	0	0	
175	1/ll*1/h	0	0	0	0	0	0	0	0	0	1	0	0	
176	1/ll*1/hh	0	0	0	0	0	0	0	0	0	0	1	0	

Thus, the final result of the direct adjustment is the determination of the vector of corrections of the measured values $\underset{51\times1}{v}$:

$$\underset{51\times1}{v} = \underset{51\times25}{A} \cdot \underset{25\times1}{x} - \underset{51\times1}{f} \tag{20}$$

The standard deviation of the measured values is thus:

$$\sigma_{xx} = \sigma_0 \cdot \sqrt{q_{xx}} = \sigma_o \cdot \sqrt{diag(Q_{xx})} \tag{21}$$

Tab. 7 *A table of the sought-after values of* q_{xx}.

SUM =takeDijagWmatrix(DX152:EV176)

	DW	DX	DY	DZ	EA	EB	EC	ED	EE	EF	EG	EH	EI	EJ	EK
151	Q_{xx}	ll*hh	ll*h	ll*0	ll*1/h	ll*1/hh	l*hh	l*h	l*0	l*1/h	l*1/hh	0*hh	0*h	0*0	0*1/h
152	ll*hh	1	0	0	0	0	0	0	0	0	0	0	0	0	0
153	ll*h	0	1	0	0	0	0	0	0	0	0	0	0	0	0
154	ll*0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
155	ll*1/h	0	0	0	1	0	0	0	0	0	0	0	0	0	0
156	ll*1/hh	0	0	0	0	1	0	0	0	0	0	0	0	0	0
157	l*hh	0	0	0	0	0	1	0	0	0	0	0	0	0	0
158	l*h	0	0	0	0	0	0	0,00012315	-4,14659E-05	0	0	0	0	0	0
159	l*0	0	0	0	0	0	0	-4,14659E-05	0,000140122	0	0	0	0	0	0
160	l*1/h	0	0	0	0	0	0	0	0	1	0	0	0	0	0
161	l*1/hh	0	0	0	0	0	0	0	0	0	1	0	0	0	0
169	1/l*0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
170	1/l*1/h	0	0	0	0	0	0	0	0	0	0	0	0	0	0
171	1/l*1/hh	0	0	0	0	0	0	0	0	0	0	0	0	0	0
172	1/ll*hh	0	0	0	0	0	0	0	0	0	0	0	0	0	0
173	1/ll*h	0	0	0	0	0	0	0	0	0	0	0	0	0	0
174	1/ll*0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
175	1/ll*1/h	0	0	0	0	0	0	0	0	0	0	0	0	0	0
176	1/ll*1/hh	0	0	0	0	0	0	0	0	0	0	0	0	0	0
177	q_{xx}	=takeDijagWmatrix(DX152:EV176)			1	1	1	0,00012315	0,000140122	1	1	1	1	1	1

Then, the inappropriate terms are all discarded, under the condition that either $a_i / \sigma_{xx} \geq \tau$, where what fits is that $\tau =$ 2nd or 3rd term of the series, or $a_i / \sigma_{xx} < \tau$, where what does not fit is that $\tau =$ lst or 2nd term of the series. Here, the coefficients $a_{i,j}$ are the opposite values of the vector of growth of the sought-after values x_{25x1}.

Tab. 8 *A depiction of the process of discarding of the terms of the*

a_i	m_{xx}		m_{xx}/a	a/m_{xx}			a_i	
2,373936176	22,08422141	ll*hh	9,30	0,1075	1		0	1
-0,406078648	5,519672063	ll*h	13,59	0,0736	1		0	1
-3,437216043	32,20727526	ll*0	9,37	0,1067	1		0	1
-0,126496702	8,220467325	ll*1/h	64,99	0,0154	1	MIN	0	1
0,68635866	6,56837211	ll*1/hh	9,57	0,1045	1		0	1
-127,5777206	1199,580264	l*hh	9,40	0,1064	1		0	1
28,99780655	310,8038571	l*h	10,72	0,0933	1		0	1
183,3545609	1722,842735	l*0	9,40	0,1064	1		0	1
7,837315559	392,1906921	l*1/h	50,04	0,0200	1		0	1
-35,45160103	338,1226636	l*1/hh	9,54	0,1048	1		0	1
2555,202637	24189,04291	0*hh	9,47	0,1056	1		0	1
-738,1822662	6907,131365	0*h	9,36	0,1069	1		0	1
-3620,088501	34316,31832	0*0	9,48	0,1055	1		0	1
-164,2080383	6931,893319	0*1/h	42,21	0,0237	1		0	1
682,7662048	6488,293079	0*1/hh	9,50	0,1052	1		0	1
-22551,40723	214379,1194	1/l*hh	9,51	0,1052	1		0	1
7939,547119	69195,73186	1/l*h	8,72	0,1147	1		0	1
31771,10352	301267,4837	1/l*0	9,48	0,1055	1		0	1
1428,22876	53901,34892	1/l*1/h	37,74	0,0265	1		0	1
-5809,245117	55005,13134	1/l*1/hh	9,47	0,1056	1		0	1
73898,23828	704148,8191	1/ll*hh	9,53	0,1049	1		0	1
-30609,84766	257027,2684	1/ll*h	8,40	0,1191	1		0	1
-103809,5703	982581,504	1/ll*0	9,47	0,1056	1		0	1
-4420,325195	155893,2002	1/ll*1/h	35,27	0,0284	1		0	1
18416,73096	173782,0067	1/ll*1/hh	9,44	0,1060	1		0	1
					25			
	m_0^2	1,4334950						
	m_0	1,197286526						

The first discarded term is $\left(s_c^2 \cdot 1/h\right)$, as the quotioent a_i / σ_{xx} takes on the smallest possible value, i.e. is smaller than $\tau = 1$ or 2. The terms are discarded in this way until the quotient does not assume the value of $\tau \geq 2$ or 3 any more, which then translates to a probability value of approximately 95.45%. Thus, 23 terms of the series are descarded under this condition and as a result, what is obtained is an expression for the calculation of genuine diagonal distance. [14] This distance can be calculated with the aid of the following expression:

$$s = a_{2,2} s_c^2 h^2 + a_{1,0} s_c \tag{22}$$

If we then take the coefficients we worked with into account, we get the following expression out of the above:

$$s = 0{,}00014 \cdot s_c^2 h^2 + 0{,}98613 \cdot s_c \tag{23}$$

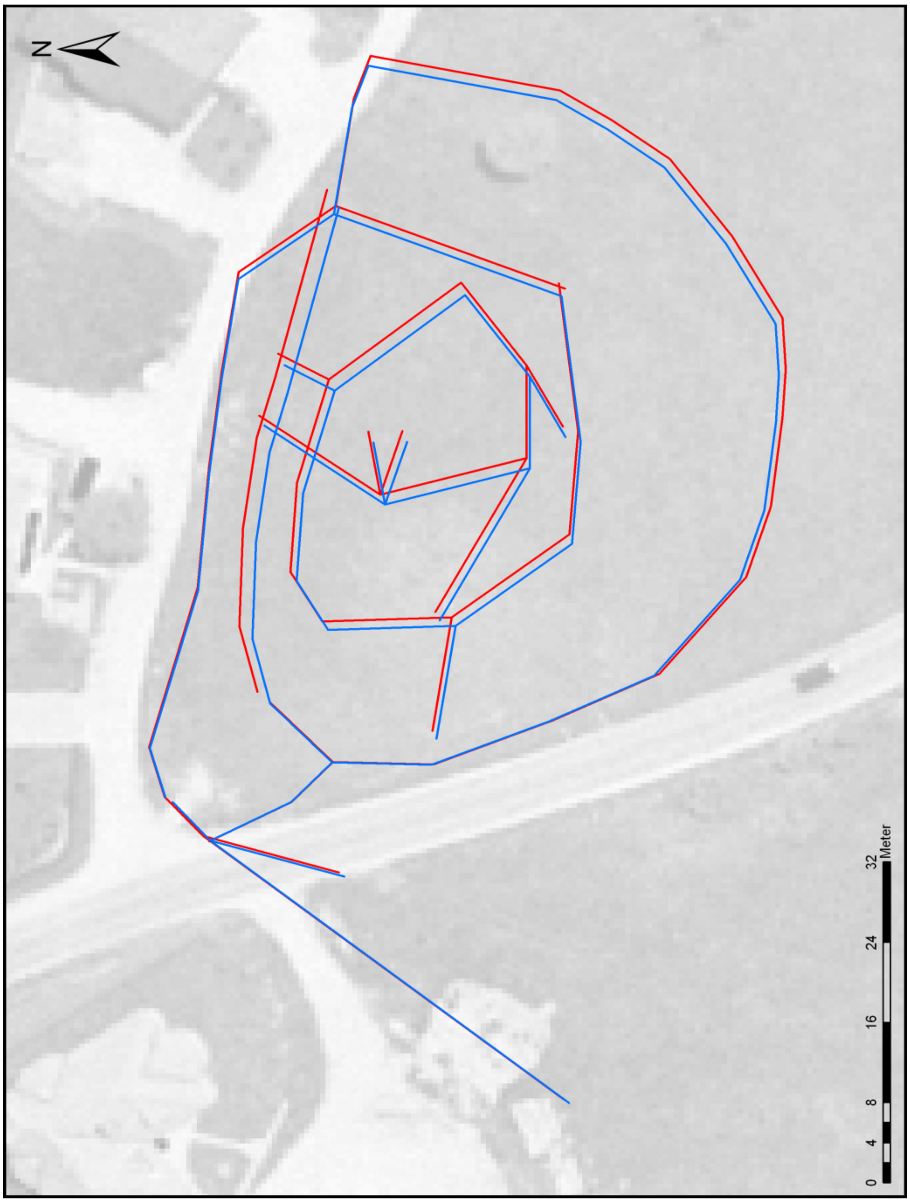

Figure 5 *The differences between the approximate and adjusted coordinates.*

With this obtained diagonal distance, we are then able to adjust the approximate coordinates [15] we have been working with and thus partially exclude the influence of the sagging of the rope or measuring tape into the shape of a catenary. The difference between these approximate and adjusted coordinates is presented with the aid of the following figure.

4 CONCLUSIONS

In an attempt to minimise its influence upon our field measurements, we have determined an appropriate approximation for a catenary curve formed out of a sagging rope or measuring tape fastened to two points of hold and stretched out over a disctance, with points of hold being either at an equal and at a different height. Now that we are aware of this fundamental approximation, it would be useful to continue with our investigation, attempting to perfect our insight into it by constantly collecting information during field observations, entering it into a common database, taking care to constantly confirm and perfect the theory behind it, and investigating the phenomenon of a sagging rope forming a catenary under different kinds of conditions.

As is evident, what may happen with a rope when it is actually used on the field as opposed to only examined in theory is that it may be different from the one that we are used to primarily analysing, meaning that it may even be cracked or uncoiling, or that conditions may not allow for us to determine certain aspects of what is happening with the curve of the rope and make them ideal, such as the strain with which the points of hold grip at the rope. In such a case, while one parameter involved in the formation of a catenary out of a sagging rope may not be easily determinable, we may still be able to evaluate other parameters and use them to help us better determine the catenary formed and how to deal with it.

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