GEOMETRY OF OVAL STRAND CREATED OF n_0+n_1+n_2 WIRES

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Abstract: *The paper deals with the mathematical geometric modelling of the oval wire strand created of $n_0 + n_1 + n_2$ wires. The mathematical representation of the wire axes is in form of parametric equations with variable input parameters. The number and diameter of the core wires are the initial data. The parametric equations of the axes of individual wires are derived based on these data and the number of wires in the layer. The equations are implemented in Pro/Engineer Wildfire V5 software for creating the geometrical model of the strand.*

Key words: *wire rope, strand of a rope, oval strand, geometrical model*

1 INTRODUCTION

Steel ropes are used in many engineering applications. Multiplicity of application requires a variousness of their structures. Design of structure affects the mechanical properties of the ropes therefore determining optimal values of their geometrical parameters plays an important role. Computer-aided design is powerful tool in this process. It allows design a geometrical construction of the rope, determine the geometric parameters and verify their relevance by creating a geometrical model [1, 2]. In the geometrical modelling of wire strands and ropes key part plays mathematical expression of the wires. Mathematical equations allow create geometrical models of the strands using the CAD system [3].

The ropes of circular cross-section constitute one group of wire ropes. These can be formed by strands of various shapes. The oval strands are one type of them. In this paper, the mathematical expression of oval strand constructed of $n_0 + n_1 + n_2$ wires is derived and their implementation to the software Pro/ENGINEER Wildfire is used to construct a geometrical model [4].

2 GEOMETRICAL CONSTRUCTION OF THE OVAL STRAND

The considered strand is made of two layers of circulary wires helically laid around a core [5]. The core consists of n_0 wires having a diameter δ_0. First layer is created by n_1 wires with diameter δ_1 and second layer is created by n_2 wires with diameter δ_2 (Fig. 1).
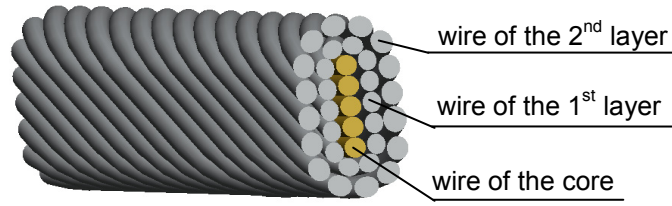
Figure 1 *Cross- section of the oval strand of $n_0 + n_1 + n_2$ type*

There is the gap Δ_j between the wires. The wires of both layers have the right- hand pitch and the winding angle is α_j.

3 MATHEMATICAL EXPRESSION OF THE WIRE AXES

  The wires helically lay around a straight core. The surface generated by the wire can be formed by translation of the circle whose center is on the wire axis and the circle lies at the normal plane of this curve. Therefore, sufficient is to derive a mathematical expression of the wire axes curves.

3.1 The wire of the first layer

  Let the right-hand Cartesian coordinate system $(O; x, y, z)$ be placed so that the z axis is identical with the axis o_S of the strand and the x axis is perpendicular to the line OX (Fig. 2).
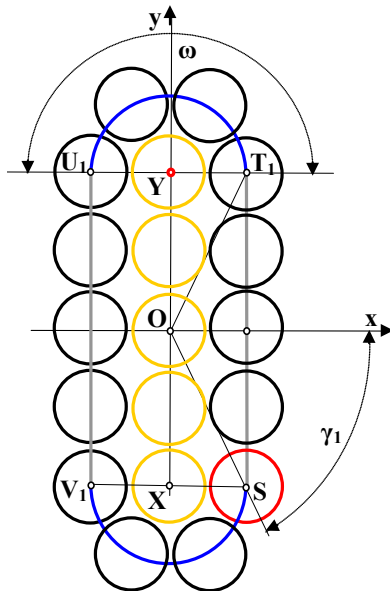

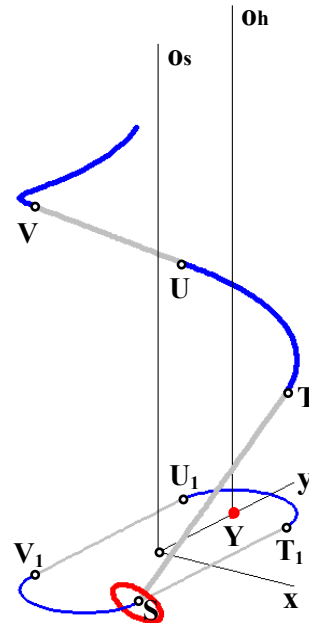**Figure 2** *Cross-section of the 1st layer*

Figure 3 *Wire axis in the 1st layer*

The curve of the wire axis consists of straight line segments and helix segments (Fig. 3). Let point S be located on the curve which will be expressed. The part ST of the curve is a line segment, the part TU is a part of cylindrical helix. We will derive the equations of both parts

using the angle of rotation around the z axis as a parameter ψ. We mark γ_1 the angle between the x axis and the line OS. The size of them is given by formula:

$$\gamma_1 = \arctan\frac{(n_0-1)(\delta_0+\Delta_0)}{\delta_0+\delta_1}. \tag{1}$$

Then the parametric equations of the line segment ST have the form:

$$x_{l_1}(\psi) = \frac{\delta_0+\delta_1}{2}, \tag{2}$$

$$y_{l_1}(\psi) = \frac{(\delta_0+\delta_1)}{2}\tan(\psi-\gamma_1), \tag{3}$$

$$z_{l_1}(\psi) = \frac{(n_0-1)(\delta_0+\Delta_0)+(\delta_0+\delta_1)\tan(\psi-\gamma_1)}{2\tan\alpha_1}, \tag{4}$$

where $\psi \in \langle 0; 2\gamma_1 \rangle$.

The part TU of the wire axis curve is cylindrical helix. Its axis o_h passing through the point Y (Fig. 3) is parallel to the z-axis. Using the transformation of the coordinate system we obtain the equations of the helix part:

$$x_{h_1}(\psi) = \frac{(\delta_0+\delta_1)}{2}\cos(\psi-2\gamma_1), \tag{5}$$

$$y_{h_1}(\psi) = \frac{(n_0-1)(\delta_0+\Delta_0)+(\delta_0+\delta_1)\sin(\psi-2\gamma_1)}{2}, \tag{6}$$

$$z_{h_1}(\psi) = \frac{(\delta_0+\delta_1)(\psi-2\gamma_1)+2(n_0-1)(\delta_0+\Delta_0)}{2\tan\alpha_1}, \tag{7}$$

where $\psi \in \langle 2\gamma_1; 2\gamma_1+\pi \rangle$.

These two segments are repeated in the curve. They are only rotated about the angle $\kappa = k\pi$ around the z axis and translate about the height

$$h_{\Delta_1} = \frac{(\delta_0+\delta_1)\pi+2(n_0-1)(\delta_0+\Delta_0)}{2\tan\alpha_1}. \tag{8}$$

3.2 The wire of the second layer

Using the previous method, we obtain a mathematical expression of the wire axis curve of second layer. For the angle between the x axis and the line OS is valid relationship

$$\gamma_2 = \arctan\frac{(n_0-1)(\delta_0+\Delta_0)}{\delta_0+2\delta_1+\delta_2}. \tag{9}$$

The line segment of the curve is expressed by the equations

$$x_{l_2}(\psi) = \frac{\delta_0+2\delta_1+\delta_2}{2}, \tag{10}$$

3

$$y_{l_2}(\psi) = \frac{(\delta_0 + 2\delta_1 + \delta_2)}{2}\tan(\psi - \gamma_2),$$ (11)

$$z_{l_2}(\psi) = \frac{(n_0-1)(\delta_0 + \Delta_0) + (\delta_0 + 2\delta_1 + \delta_2)\tan(\psi - \gamma_2)}{2\tan\alpha_2},$$ (12)

where $\psi \in \langle 0; 2\gamma_2 \rangle$.

The helix segment of the curve can be expressed by the forme

$$x_{h_2}(\psi) = \frac{(\delta_0 + 2\delta_1 + \delta_2)}{2}\cos(\psi - 2\gamma_2),$$ (13)

$$y_{h_2}(\psi) = \frac{(n_0-1)(\delta_0 + \Delta_0) + (\delta_0 + 2\delta_1 + \delta_2)\sin(\psi - 2\gamma_2)}{2},$$ (14)

$$z_{h_2}(\psi) = \frac{2(n_0-1)(\delta_0 + \Delta_0) + (\delta_0 + 2\delta_1 + \delta_2)(\psi - 2\gamma_2)}{2\tan(\alpha_2)},$$ (15)

for $\psi \in \langle 2\gamma_2; 2\gamma_2 + \pi \rangle$. The translation of the segment in the curve is

$$h_{\Delta_2}(\psi) = \frac{2(n_0-1)(\delta_0 + \Delta_0) + (\delta_0 + 2\delta_1 + \delta_2)\pi}{2\tan(\alpha_2)}.$$ (16)

3.3 Other wires of the layer

Wires of one layer are the same surfaces. This fact can be used. Each wire in the layer is given by the previous one shifted by a particular size h_w in the axial direction. The size is depended on the one pitch length h_j and the number n_j of wires in the layer j. It can be calculated from the relationship:

$$h_{w_j} = \frac{h_j}{n_j}.$$ (17)

So, the curve of any wire axis of the layer can be expressed by parametric equations

$$x(\psi) = x_s(\psi)\cos\kappa - y_s(\psi)\sin\kappa,$$ (18)

$$y(\psi) = x_s(\psi)\sin\kappa + y_s(\psi)\cos\kappa,$$ (19)

$$z(\psi) = z_s(\psi) + kh_{\Delta_j} + ih_{w_j},$$ (20)

in which for the first layer we use the equations (1) - (4), $\psi \in \langle 0; 2\gamma_1 \rangle$ for line segment ($s = l_1$) and the equations (5) – (7), $\psi \in \langle 2\gamma_1; 2\gamma_1 + \pi \rangle$ for helical segment ($s = h_1$). For the second layer we use the equations (9) – (12) for $s = l_2$ and (13) – (15) for $s = h_2$.

4 MODELLING OF THE WIRES IN THE LAYER

Based on the mathematical expression it is possible to construct the geometrical model of the wires and subsequently of the strand. To illustrate this possibility the models of the strands with 4+10+13 wires and 4+10+14 wires are constructed. In this case the parametric equations are implemented in Pro/ENGINEER Wildfire software for the geometric modelling.

Let us assume, that the diameter of the core wires, the gap between them and the winding angles of both layers are given. The strands selected to illustrate differ only by the number of wires in the second layer. Diameters and gaps for the layers must be calculated. It is possible on the base of the derived equations. Basic geometrical parameters of the strands are listed in Table 1.

Table 1 *Basic parameters of the strand of type* 4+10+13 *and type* 4+10+14

Strand of 4+9+13 wires	Core	First layer	Second layer	
Number of the wires n_j	4	10	13	14
Winding angle α_j (°)	-	11,00	11,00	11,00
Diameter δ_j of the wire (mm)	1,18	1,446	1,941	1,749
Gap Δ_j between wires (mm)	0,00	0,087	0,076	0,062

Geometrical model of the strand with 4+10+13 wires is shown in Figure 4.
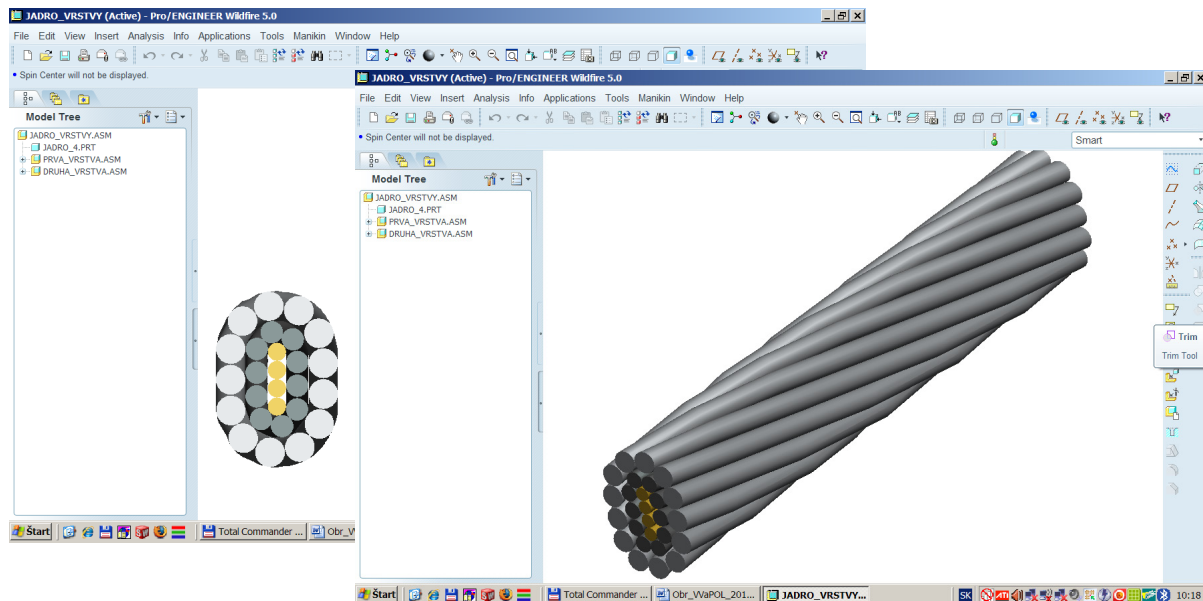

Figure 4 *Geometrical model of the strand of* 4+10+13 *wires*

Geometrical model of the strand of 4+10+14 wires compared to the previous changes only in the second layer, which has a different number of wires (Fig. 5).
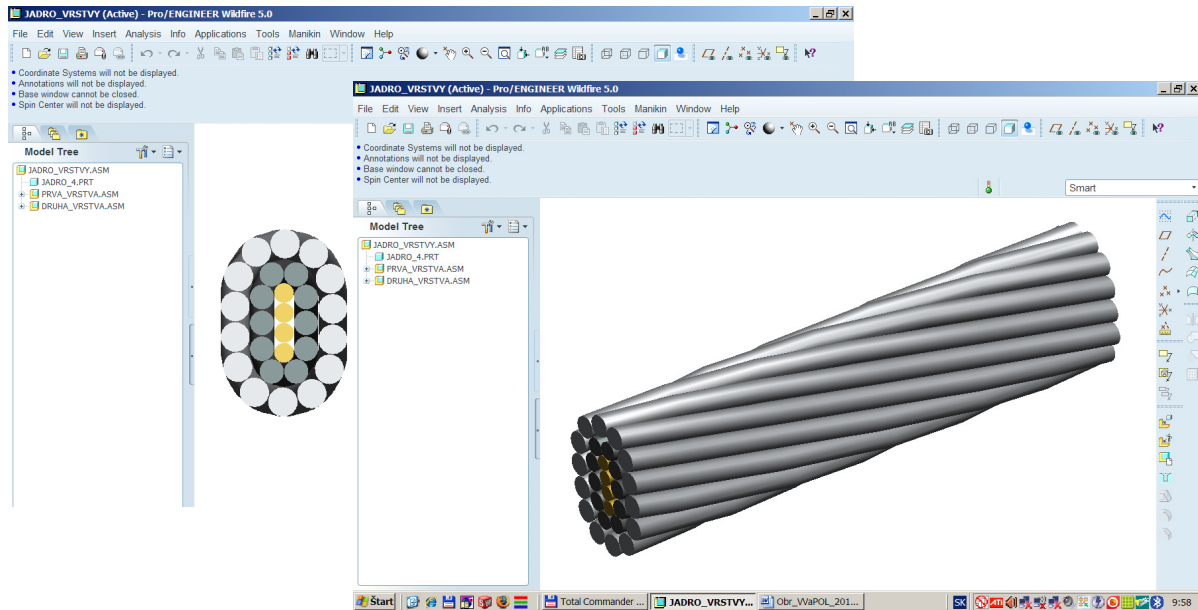
Figure 5 *Geometrical model of the strand of* 4+10+14 *wires*

5 CONCLUSION

In order to create the geometrical model of the oval strand, the parametric equations have been developed and implemented in Pro/ENGINEER Wildfire V5 software for the modelling.

Developed equations allow us to create the model of oval strand with two layers and the core consisting of n_0 wires. Based on the geometric parameters that define the core, we can to determine necessary parameters for any given number of wires in the layer.

The strands selected to illustrate differed by the number of wires in the second layer. The parametric equations with concrete input parameters were implemented in the said software and geometrical models of the wires and consequently two strands were created.

The described method of the creation of geometric model of oval strand can be used to explicit computer modelling and analysis of wire strands and ropes.

Acknowledgements

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