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A SOLUTION OF MOVING CONVEYOR BELT CRITICAL VELOCITY PROBLEM OF THE FREE TRANSVERSE VIBRATIONS

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Key words: Conveyor belt, critical velocity, moving belt, free transverse vibration

Abstract:

For the safe and economic engineering design of conveyor belt systems, it is essential to study and determine the critical velocities given by the eigenvalues of the moving conveyor belts subjected to constant axial force. As in many other vibration characteristics, the eigenvalues and related natural frequencies of the belt under investigation also depend on the magnitude of the belt velocity and the axial force. For any value of the constant axial force, natural frequencies gradually decrease when the belt velocity increase and become zero for some velocity values which are defined as critical velocities. In this paper, the critical velocities of free vibrations of conveyor belts have been investigated analytically when the belt is fixed or simply supported at both ends. For the visual representation of the results, frequency values as a function of belt velocity and longitudinal force values are also given in graphical forms.

1. Introduction

Axially moving systems are very common in engineering problems which arise in industrial, civil, aerospace, mechanical, electronic and automotive applications. Aerial cables, tram-ways, oil pipe lines, magnetic tapes, power transmission belts, paper sheet and web processes, fiber winding and band saw blades are instances for cases where an axial transport of mass can be associated with transverse vibrations. Investigating transverse vibrations of a belt system is a challenging subject which has been studied for many years and is still of interest today [23].

Ashley&Haviland investigated the bending vibrations of the pipe lines containing fluids. Free vibrations and forced oscillations which occur because of the lateral winds can cause crucial problems in the design of the pipe lines that are fixed to the ground. In order to solve these problems, beam theory can be used. Regarding this analytical method, it is observed that when the velocity of the flow is low, pipe lines are not subjected to vibrations and the critical velocities due to high fluid velocities cause dynamic stability [22].

For linear vibration analysis of an axially moving string, Wickert and Mote modified the classical analysis method and got the response in closed forms for arbitrary excitation and initial conditions. Moon and Wickert improved a modal perturbation solution in the context of the asymptotic method of Krilov–Bogoliubov–Mitropolsky for nonlinear forced oscillations of moving materials [21].

Mote has investigated approximate stability-instability region boundaries for two cases of parametric excitation. The first problem deals with periodic, axial, tension variation of slender, axially moving materials such as band saws, belts, tapes, strings and chains; the second case considers instability caused by periodic, in-plane, edge loading in axially moving materials[21].

In this study, vibration of belts with flexural rigidity and uniform motion is investigated [1].The structure of the problem is related to the type of mechanical systems which is commonly referred to as

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axially moving continua [2]. The tensioned and traveling Euler-Bernoulli beam is a common example of moving materials. The analysis of the vibration and dynamic stability characteristics of such systems are important for the optimal design.

The eigenvalues of the moving belt is related to the magnitude of the constant belt velocity and axial force. The natural frequency of each mode decreases with the increasing belt speed. There exists some combination of the velocity and force values where the natural frequencies of the moving belt are equal to zero. The velocity values which make the natural frequencies zero are called the critical velocities. At sufficiently high speed near critical velocities the belt experiences instability. In general, in addition to resonance due to direct excitation, an axially moving belt may exhibit a standing wave (or divergence buckling) type instability at critical speed, a dynamic (or flutter) type instability at speeds above the critical one, and Mathieu type instability due to parametric excitation [2, 3].

In conveyor applications, the belt can exhibit some combination of torsional, longitudinal, and transverse vibrations. In this study, only transverse belt vibration is investigated since the natural frequencies associated with torsional and longitudinal vibrations are quite high for typical belt geometries, materials, and conveyor design geometries. In this paper, a proper model for the moving belt's vibration analysis has been presented and especially the critical velocity values obtained for the boundary conditions of both the fixed and simply supported ends. This work presents the stability ranges near critical velocities and provides the general conclusion on conveyor belt design.

The method presented in this paper can be adequate in the design of conveyor belts for practical considerations on the avoidance of the critical velocity ranges. However, it is clearly seen that the method in this paper is simple and straightforward.

2. Basic Equations

The governing equation of motion of a moving elastic belt is analogous to those of power transmission belts, band-saw blades, and pipelines containing flowing fluids [4-8]. In this regard, investigations about this subject have been studied. For instance, Ashley&Haviland investigated the bending vibrations of the pipe lines containing fluids. Free vibrations and forced oscillations which occurs because of the lateral winds can cause crucial problems in the design of the pipe lines that are fixed to the ground. In order to solve these problems, beam theory can be used. Regarding this analytical method, it is observed that when the velocity of the flow is low, pipe lines are not subjected to vibrations and the critical velocities which occur because of high fluid velocities cause dynamic stability.[23]

Fig. 1 depicts the dynamic and geometric model of the moving conveyor belt supported on two adjacent rollers. The Xw coordinate system where the motion is observed is fixed to the belt. Let v be the axial velocity of the belt, S be the axial force applied to the belt, ρ be the mass density of the belt, F be the cross-sectional area of the belt, E be the Young's modulus, J be the centroidal moment of inertia of the belt's cross-sectional area, and $w(X,T)$ be the transverse deflection of the belt, measured from its equilibrium position.

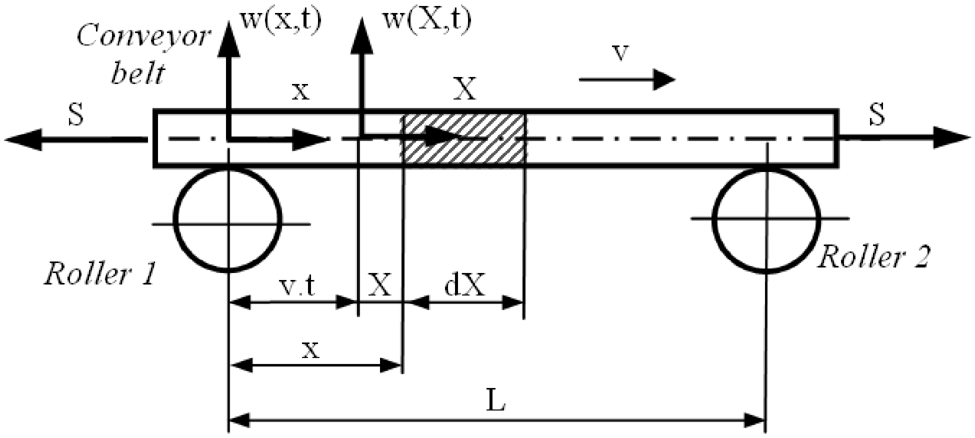

Fig. 1: *The moving conveyor belt supported on two adjacent rollers*

Free-body diagram of an arbitrary differential element of the belt at an arbitrary instant of time is shown in Fig. 2. By assuming the lateral deflections and slopes to be small, the change in tension S with deflection is negligible and can be ignored. The internal normal force which is the resultant of the normal stress distribution is equal to the tension S. The internal bending moment is the resultant moment of the normal stress distribution and the internal shear force is the resultant of the shear stress distribution. The effective force is equal to the element mass multiplied by its acceleration. The element's axial and angular accelerations are assumed to be small in comparison to other effects and are thus ignored.

The equation of transversal motion in *w*-direction is

$$V - \left(V + \frac{\partial V}{\partial X} dX \right) + Sq - S\left(q + \frac{\partial q}{\partial X} dX \right) = rF\frac{\partial^2 w}{\partial T^2} dX \quad (1)$$

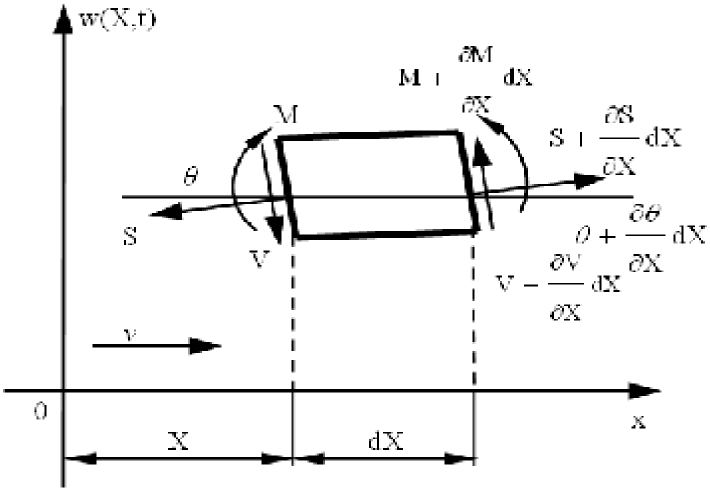

Fig. 2: *Free-body diagram of an arbitrary differential element of the belt at an arbitrary instant of time*

The slope of the differential element is given by $q = \partial w / \partial X$. The equation of rotational motion about the neutral axis at the left face of the element can be written as

$$M - \left(M + \frac{\partial M}{\partial X} dX \right) + \left(V + \frac{\partial V}{\partial X} dX \right) dX = rF\frac{\partial^2 w}{\partial T^2} dX \left(\frac{dX}{2} \right). \quad (2)$$

Since dX is infinitesimal, terms of order dX^2 and the moment of S about O are negligible when compared to terms of order dX . Thus Eq.(2) is reduced to $V = \partial M / \partial X$. From mechanics of materials, one finds

$$M = EJ\frac{\partial^2 w}{\partial X^2}. \quad (3)$$

The governing differential equation of motion in the w -direction is

$$EJ\frac{\partial^4 w}{\partial X^4} - S\frac{\partial^2 w}{\partial X^2} + rF\frac{\partial^2 w}{\partial T^2} = 0. \quad (4)$$

By considering the Galilean transformation, the governing equation of motion relative to the reference frame attached to the earth can be obtained as [12, 13]

$$EJ\frac{\partial^4 w}{\partial x^4} + (rFv^2 - S)\frac{\partial^2 w}{\partial x^2} + 2vrF\frac{\partial^2 w}{\partial x \partial t} + rF\frac{\partial^2 w}{\partial t^2} = 0, \quad (5)$$

where the mass per unit length rF , and the flexural rigidity EJ , the axial velocity v , the axial tension S are all assumed to be constant along the belt for the analysis of the dynamic behavior of the model in hand. The differential equation of motion (5) represents the dynamic behavior of the axially moving belt under the assumption of small transverse deflections. It must be noted that the linear analysis can be applied to this small transverse displacement equation. In the model presented here, rotary inertia and shear deformation effects are ignored since it is assumed that their cross-sectional dimension of the belt is small compared to the distance between the two adjacent rollers, the belt span, and/or the frequencies are low [9,14-16]. In Eq.(5), the second term represents the combination of the centripetal and axial force, the third term represents the Coriolis force caused by the simultaneous effects of the linear and rotational motions and, the last term represents the centrifugal force related to the instantaneous curvature.

The nondimensional form of the governing equation reads

$$\frac{\partial^4 h}{\partial x^4} + b\,\frac{\partial^2 h}{\partial x^2} + 2a\,\frac{\partial^2 h}{\partial x \partial t} + \frac{\partial^2 h}{\partial t^2} = 0\,, \tag{6}$$

where

$$x = \frac{x}{L}, \quad h = \frac{w}{L}, \quad t = \frac{\sqrt{EJ/rF}}{L^2} t \text{ and}$$

and the coefficients used are

$$a = \frac{vL}{\sqrt{EJ/rF}}, \quad b = \frac{L^2}{EJ}\left(rFv^2 - S\right), \quad b = a^2 - \frac{L^2}{EJ}S\,.$$

Thus the problem reduces to solving the Eq.(6) subject to prescribed boundary conditions. The response of the model to general excitation and initial conditions cannot be analytically predicted and understanding of the general response of axially moving materials is limited [10, 11, 17]. The accuracy of the model depends on the band velocity and tension. Since the effect of tension variation during vibration becomes increasingly significant with the increase in the axial velocity the effect of nonlinear terms should be considered for the accurate analysis of the oscillation at high belt speeds [18-20].

3. Critical Velocities

The nondimensional form of the differential equation of motion, Eq.(6), for the problem at hand are partial differential equation of the forth order. The problem thus reduces to solving the above differential equation subject to prescribed boundary conditions. The method of the separation of time and space variables can be applied to seek a solution to the problem. Let the partial differential equation in Eq.(6) be satisfied by the functions of the form

$$h(x,t) = y(x)\cdot e^{wt}\,, \tag{7}$$

where the eigenfunction $y(x)$ is a function of nondimensional space variable x alone and ewt a function of nondimensional time variable t alone. Substituting Eq.(7) for h in the differential Eq.(6) yields

$$\frac{d^4 y}{dx^4} + b\,\frac{d^2 y}{dx^2} + 2aw\,\frac{dy}{dx} + wy = 0\,. \tag{8}$$

This is a homogeneous linear ordinary differential equation of fourth order with constant coefficients. For the solution of Eq.(8), one can find

$$y(x) = C \cdot e^{rx}\,, \tag{9}$$

where C and r are constants. Since the Eq.(8) is a fourth order linear differential equation, there exists C_i and r_i for $i = 1, 2, 3, 4$ to be determined. The constants r_i are determined as the roots of the characteristic equation of the differential Eq.(8) which is

$$r^4 + b \cdot r^2 + 2a \cdot w \cdot r + w^2 = 0\,, \tag{10}$$

and the constants C_i can be found from the boundary conditions since the solvability condition is determined by applying the homogeneous boundary conditions to the eigenfunction $y(x)$. For the belts fixed at both ends, the homogeneous boundary conditions are:

$$y(0) = y(1) = 0 \quad \text{and} \quad \left.\frac{dy}{dx}\right|_{x=0} = \left.\frac{dy}{dx}\right|_{x=1} = 0 \tag{11}$$

For the simply supported belts at both ends, the homogeneous boundary conditions can be stated as

$$y(0) = y(1) = 0 \ \text{and} \ \left.\frac{d^2y}{dx^2}\right|_{x=0} = \left.\frac{d^2y}{dx^2}\right|_{x=1} = 0 \tag{12}$$

These conditions are homogeneous since they do not involve functions of t. Therefore, application of the boundary conditions to the solution (9) will yield four homogeneous algebraic equations for C_i. The condition for the existence of a nontrivial solution is that the coefficient determinant is zero. This gives an equation for the determination of w, which is the frequency equation of the form $F(a, b, w) = 0$. The solution of the frequency equation in terms of a and b implicitly defines $w = w(a, b)$.

The typical result is that as the nondimensional velocity increases from zero, the frequencies of each mode gradually decrease until they become zero. The velocities for which the frequencies become zero are called critical velocities, as stated before.

3.1. Critical Velocities for Fixed Ends.

When the boundary condition for fixed-fixed ends is of practical interest it fortunate that the critical velocities can be determined without obtaining the complete solution. For $w = 0$ the differential Eq.(8) becomes

$$\frac{d^4y}{dx^4} + b \cdot \frac{d^2y}{dx^2} = 0. \tag{13}$$

Eq.(13) has a solution of the form of Eq.(9). The substitution Eq.(9) into Eq.(13) leads to the characteristic equation and also the general solution. Thus the characteristic equation is

$$r^4 + b \cdot r^2 = 0 \ \text{ or } \ r^2(r^2 + b) = 0 \ , \tag{14}$$

and the roots of differential equation are determined as follows

$$r_1 = r_2 = 0, \quad r_3 = i \cdot \sqrt{b} \ , \text{ and } \ r_4 = -i \cdot \sqrt{b} \ . \tag{15}$$

With a double root at zero, the general solution to Eq.(13) is

$$y = C_1 + C_2 \cdot x + C_3 \cdot e^{i g x} + C_4 \cdot e^{-i g x} \ , \tag{16}$$

where $\sqrt{b} = g$.

Applying the boundary conditions (11) for fixed-fixed ends to Eq.(16) yields four equations in C_1, C_2, C_3, and C_4,

$$\begin{aligned} &C_1 + C_3 + C_4 = 0 \\ &C_2 + i \cdot g \cdot C_3 - i \cdot g \cdot C_4 = 0 \\ &C_1 + C_2 + e^{ig} \cdot C_3 + e^{-ig} \cdot C_4 = 0 \\ &C_2 + i \cdot g \cdot e^{i \cdot g} \cdot C_3 - i \cdot g \cdot e^{-i \cdot g} \cdot C_4 = 0 \end{aligned} \tag{17}$$

The nontrivial solution for C_1, C_2, C_3, and C_4 requires that the determinant of their coefficients be zero, that is,

$$\Delta(g) = \begin{vmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & i \cdot g & -i \cdot g \\ 1 & 1 & e^{i \cdot g} & e^{-i \cdot g} \\ 0 & 1 & i \cdot g \cdot e^{i \cdot g} & -i \cdot g \cdot e^{-i \cdot g} \end{vmatrix} = 0. \quad (18)$$

Expanding the determinant gives

$$\cos(g) + \frac{g}{2} \cdot \sin(g) - 1 = 0. \quad (19)$$

The transcendental Eq.(19) can be solved by applying appropriate numerical methods, such as the bisection algorithm. After some arrangement, the dimensional critical velocities for the first eight modes as a function of the axial forces from a sample problem are shown in Fig. 3 when the ends are fixed.

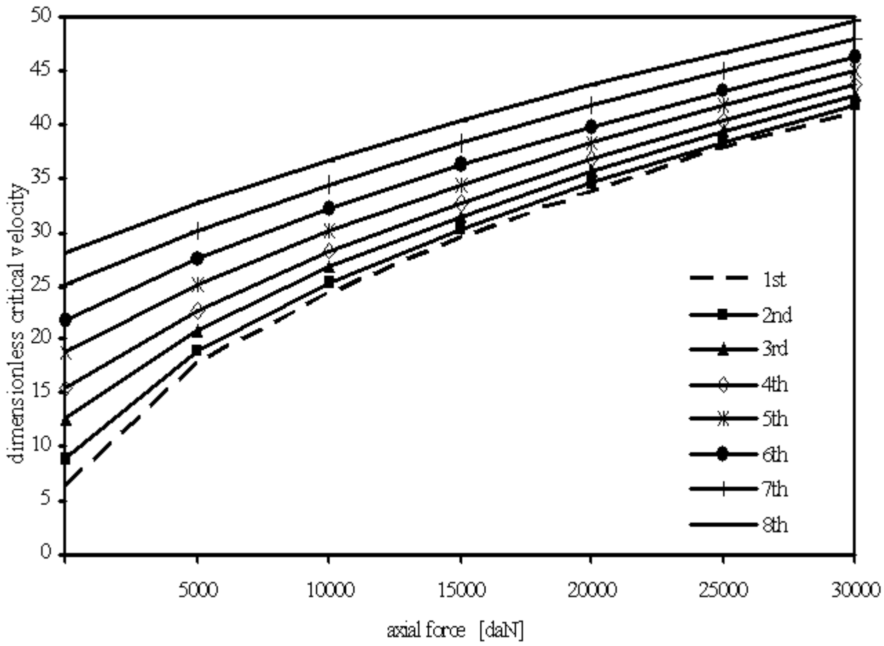

Fig. 3*: The dimensionless critical velocities for fixed ends.*

3.2. Critical Velocities for Simply Supported Ends

The application of the boundary conditions (12) for simply supported ends to the general the general solution, equation (16) of the differential Eq.(13) yields

$$\begin{aligned} &C_1 + C_3 + C_4 = 0 \\ &C_3 + C_4 = 0 \\ &C_1 + C_2 + e^{ig} \cdot C_3 + e^{-ig} \cdot C_4 = 0 \\ &e^{i \cdot g} \cdot C_3 + e^{-i \cdot g} \cdot C_4 = 0 \end{aligned} \quad (20)$$

The system of Eqs.(10) has a nontrivial solution if and only if the determinant of coefficients is zero, that is,

$$\Delta(g) = \begin{vmatrix} 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & e^{i\cdot g} & e^{-i\cdot g} \\ 0 & 1 & e^{i\cdot g} & e^{-i\cdot g} \end{vmatrix} = 0 \quad (21)$$

Expanding this determinant and rearranging yield $\sin g = 0$.The dimensionless values of critical velocities will be determined from the solution of this equation, which is $g = kp \quad (k = 1, 2, 3, \mathbf{L}\ )$.

The nondimensional critical velocities for the first eight modes as a function of the axial forces from a sample problem are shown in Fig. 4 when the ends are simply supported.

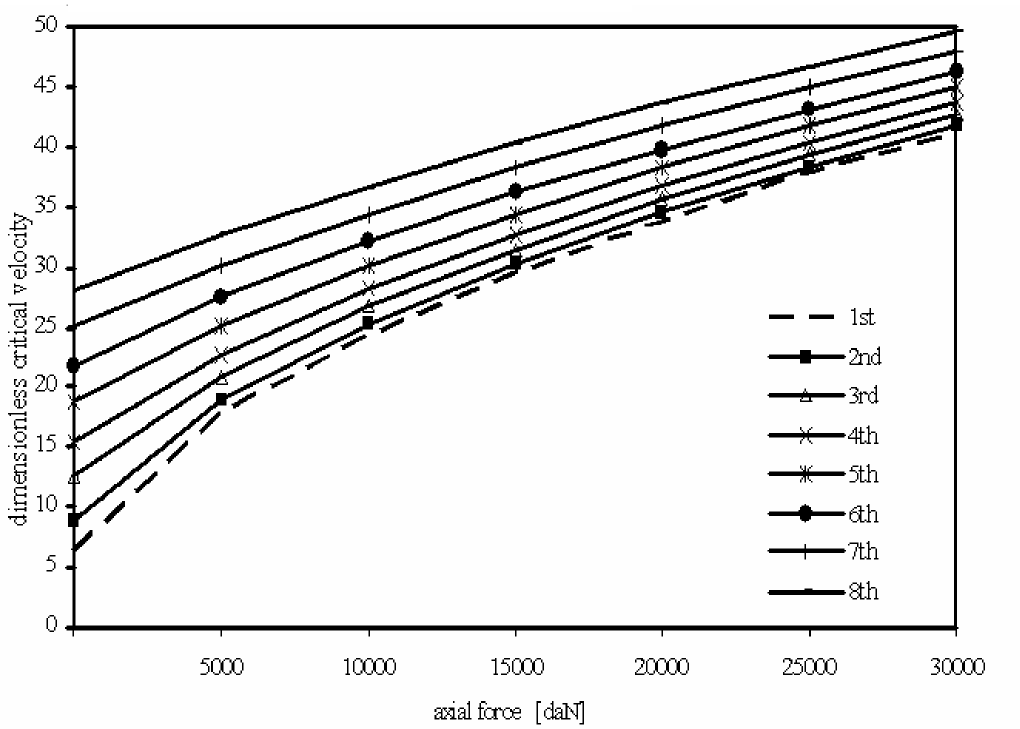

Fig. 4: *The dimensionless critical velocities for simply supported ends*

In general the natural frequencies of each mode decrease monotonically as the belt velocity increase. When the frequency of a mode vanishes the critical velocity of that mode is obtained. Therefore the belt has many critical speeds and it is not recommended to run the belt near these critical velocities as the instabilities or resonance effect may occur in the system.

4. Conclusion

This paper analyses the critical velocities of free vibrations of conveyor belts analytically when the belt is fixed or simply supported at both ends. It has been determined that the axial traction force has some effects on the amplitudes of the modes When the axial force increases it has been observed that the amplitudes decrease but the natural frequencies and the critical velocities increase. The present study deals with a linear system experiencing small amplitude transverse vibrations and ignores the effects of the rotary inertia and shear deformation. The method presented in this paper can be adequate in the design of conveyor belts for practical considerations on the avoidance of the critical velocity ranges. However, it is clearly seen that the method in this paper is simple and straightforward.

References:

[1] S. Arbrat, Vibration of a Moving Belt in: Proceeding of 1st International Applied Mechanical System Design Conference, 1989.

[2] C.D. Mote, Dynamic Stability of Axially Moving Materials, Shock and Vibration Digest Vol. 4, pp. 2-11, 1972.

[3] A.G. Ulsoy, J.E. Whitesell and M.D. Hooven, Design of Belt-tensioner Systems for Dynamic Stability, Trans. of the ASME, Journal of Vibration, Acoustics, Stress and Reliability in Design Vol. 107, pp. 282-290, 1985.

[4] Y.L. Kwon, and J.G. Ih, Vibrational Power Flow in the Moving Belt Passing Through a Tensioner, Journal of Sound and Vibration Vol. 229, pp. 329-353, 2000.
[5] M.G. Yue, Belt Vibration Considering Moving Contact and Parametric Excitation, Trans. of the ASME, Journal of Mechanical Design Vol. 115, pp. 1024-1030, 1993.
[6] C.D. Mote and S. Naguleswaran, Theoretical and Experimental Band Saw Vibrations, Trans. of the ASME, Journal of Engineering for Industry Vol. 88, pp. 151-156, 1966.
[7] S. Naguleswaran and C.J.H. Williams, Lateral Vibration of Band-saw Blades, Pulley Belts and the Like, Int. Journal of Mechanical Sciences, Vol. 10, pp. 239-250, 1968.
[8] R.H. Long, Experimental and Theoretical Study of Transverse Vibration of a Tube Containing Flowing Fluid, Trans. of the ASME, Journal of Applied Mechanics Vol. 77, pp. 65-68, 1955.
[9] A.D. Dimarogonas and S. Haddad, Vibration for Engineers, Prentice Hall, New Jersey, 1992.
[10] J.A. Wickert and C.D. Mote, Current Research on the Vibration and Stability of Axially Moving Materials, Shock and Vibration Digest Vol. 20, pp. 3-13, 1988.
[11] J.A. Wickert and C.D. Mote, Classical Vibration Analysis of Axially Moving Continua, Trans. of the ASME, Journal of Applied Mechanics Vol. 57, pp. 738-744, 1990.
[12] A.A.N. Al-Jawi, C. Pierre and A.G. Ulsoy, Vibration Localization in Dual-span, Axially Moving Beams, Part I: Formulation and Results, Journal of Sound and Vibration Vol. 179, pp. 243-266, 1995.
[13] I. Gerdemeli, The Free Transverse Vibrations of the Moving Bands Subjected to Axial Force, PhD. Thesis, Istanbul Technical University, Turkey, 1990.
[14] S.S. Rao, Mechanical Vibrations, Addison-Wesley, New York, 1995.
[15] W.T. Thomson, Theory of Vibration with Applications, Prentice Hall, New Jersey, 1988.
[16] F.S. Tse, I.E. Morse and R.T. Hinkle, Mechanical Vibrations: Theory and Applications, Allyn and Bacon, Boston, 1978.
[17] S. Chonanc, Steady State Response of an Axially Moving Strip Subjected to a Stationary Lateral Load, Journal of Sound and Vibration Vol. 107, pp.155-165, 1986.
[18] C.D. Mote, On the Nonlinear Oscillation of an Axially Moving String, Trans. of the ASME, Journal of Applied Mechanics Vol. 33, pp. 463-464, 1966.
[19] J. Moon and J.A. Wickert, Non-linear Vibration of Power Transmission Belts, Journal of Sound and Vibration Vol. 200, pp. 419-431, 1997.
[20] A.L. Thurman and C.D. Mote, Free, Periodic, Nonlinear Oscillation of an Axially Moving String, Trans. of the ASME, Journal of Applied Mechanics Vol. 36, pp. 83-91, 1969.
[21] Katica (Stevanovi´c) Hedrih, Transversal vibrations of the axially moving sandwich belts, Arch Appl. Mech 77: 523–539, 2007.
[22] Ashley,H. and Haviland, G.,Bending Vibrations of a Pipe Line Containing Flowing Fluid, Journal of Applied Mechanics, Trans. ASME, Vol. 74, pp. 205-208, 1952.
[23] On the Weakly Nonlinear, Transversal Vibrations of a Conveyor Belt with a Low and Time-Varying Velocity.

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