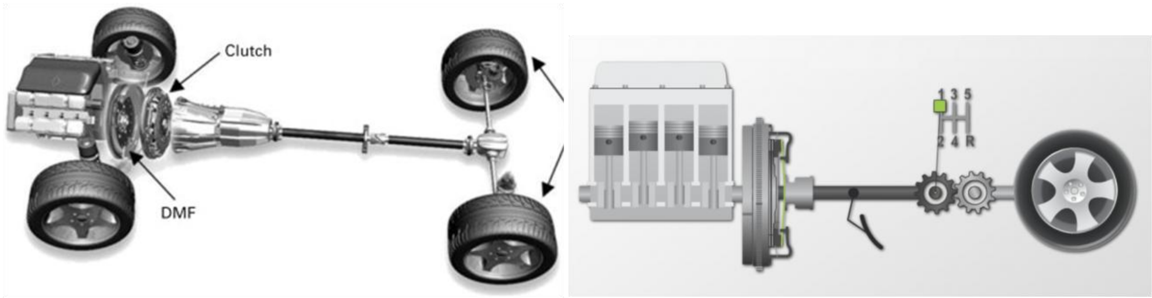
Torsional vibrations and Dual mass flywheel

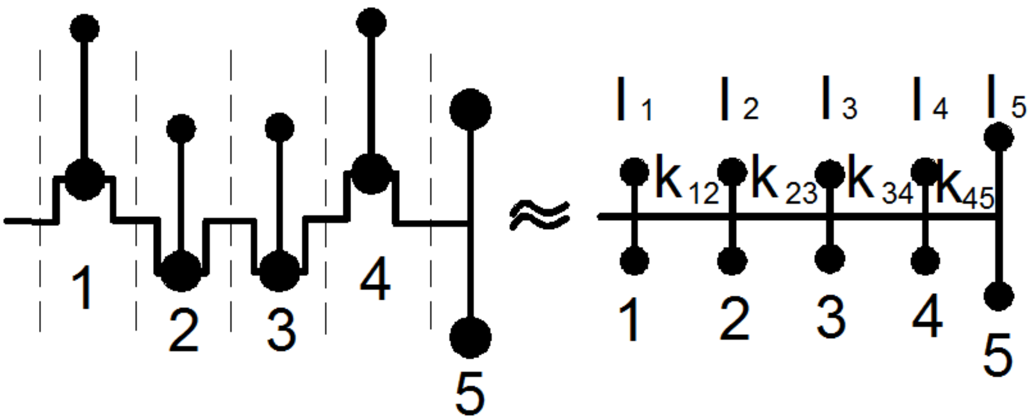
Reciprocating internal combustion engines represent the group of the most significant torsional vibration exciters. In order to be able to use analytical procedures to calculate the torsional vibration, it is necessary to simplify the complex crankshaft system and neglect the nonlinearity in the system for the first approximations.



The calculation procedure is as follows:

1. Reduction of complex shapes of bodies to simple rotating disks with a certain moment of inertia which are connected by torsional stiffness.

1a) Mass reduction
- the principle of conservation of kinetic energy



Flywheel side mass reduction.
The moment of inertia on the flywheel side is given by the equation:

$$I_{celk\_zotr} = I_{zotrva} + I_{kluka\_zotrva} + I_{or} \ [kg.m2]$$

If:
$I_{zotrva}[kg.m^2]$ - moment of inertia on the flywheel
$I_{kluka\_zotrva}[kg.m^2]$ - moment of inertia on part of crankshaft.

1b) Length reduction
The reduction in crankshaft lengths is calculated from the potential energy so that the potential energy caused by the actual bend twist is the same as the potential energy of the spare wheel. This equality can be expressed by the equation:

$$\frac{1}{2}.\frac{G.J_p}{l}.\varphi^2 = \frac{1}{2}.\frac{G.J_{p\,red}}{l_{red}}.\varphi^2$$

when
G [*MPa*] - modulus of elasticity in shear
Ip [*m*4] - polar quadratic moment of inertia of the cross section
l [*m*] - length of twisted cross section.

The reduced values of J_{pred} and l_{red} correspond to the calculate (surrogate) model.
Then it applies to the reduced length:

$$l_{red} = \frac{J_{pred}}{J_p}.l$$

As with mass reduction, certain simplified assumptions are envisaged here as well. One of them is the consideration of twisting of the 1st kind. It is possible to use other models, but these were mostly created on the basis of actually produced and used crankshafts. The most accurate method would be if torsional stiffness were measured experimentally. This would mean the physical production of the crankshaft.
Reduced bend length according to **Ker-Wilson**.
The reduced bend length is calculated using the equation:

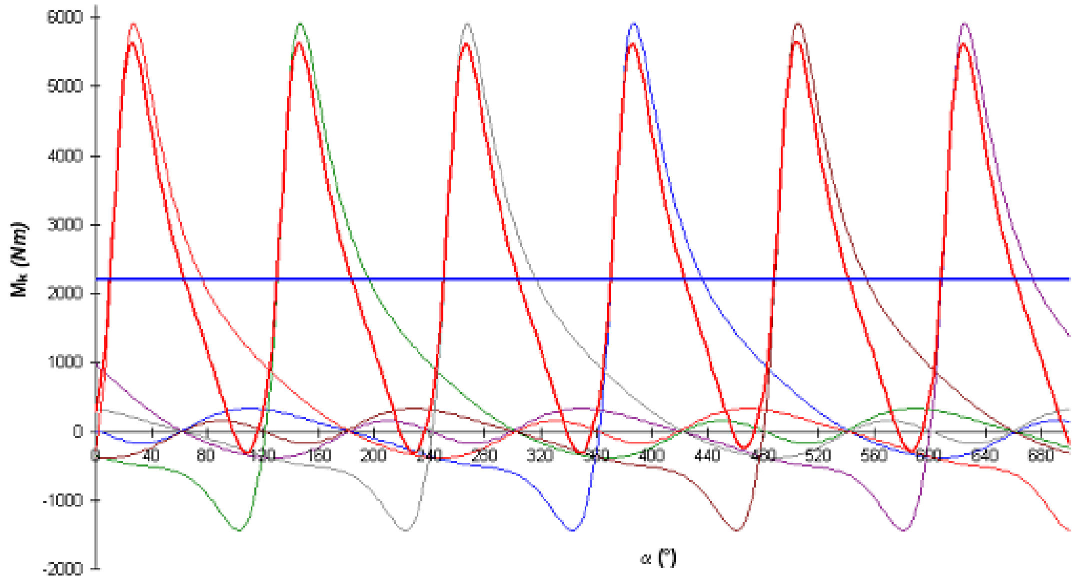
$$l_{red} = D_{red}^4 \cdot \left[\frac{b_{hc} + 0.4 \cdot D_{hc}}{D_{hc}^4} + \frac{b_{kc} + 0.4 \cdot D_{kc}}{D_{kc}^4} + \frac{r - 0.4 \cdot (D_{hc} + D_{kc})}{b \cdot h^3}\right] [m]$$

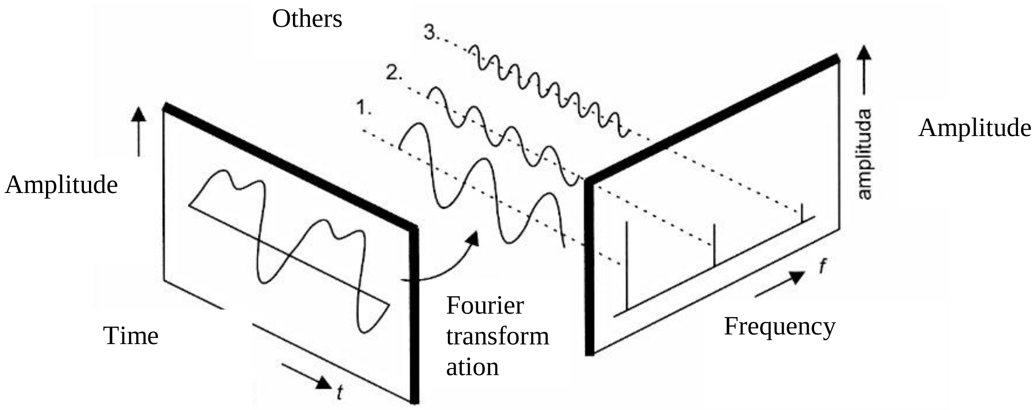
When

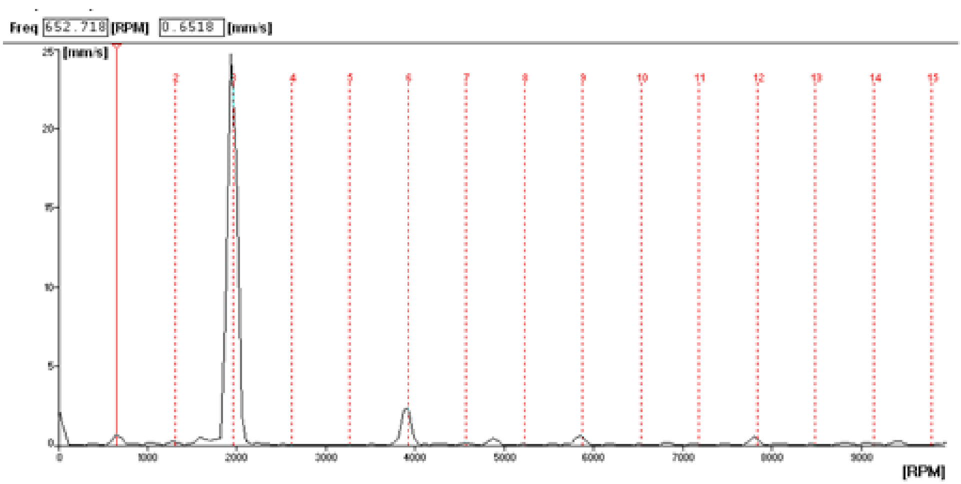
Dred [*m*] - reduced diameter, usually chosen in accordance with the diameter of the main pin of crankshaft.
Dhc [*m*] – diameter of main pin
Dkc [*m*] – diameter of pin of connecting rod
bhc [*m*] – width of main pin
bkc [*m*] – width of connecting rod pin
h [*m*] - bend arm width
b [*m*] - bending arm thickness

2. Determination of natural and forced oscillations at different oscillation shapes and resonances from harmonic components of torque in the operating area.

2.1 Load torque

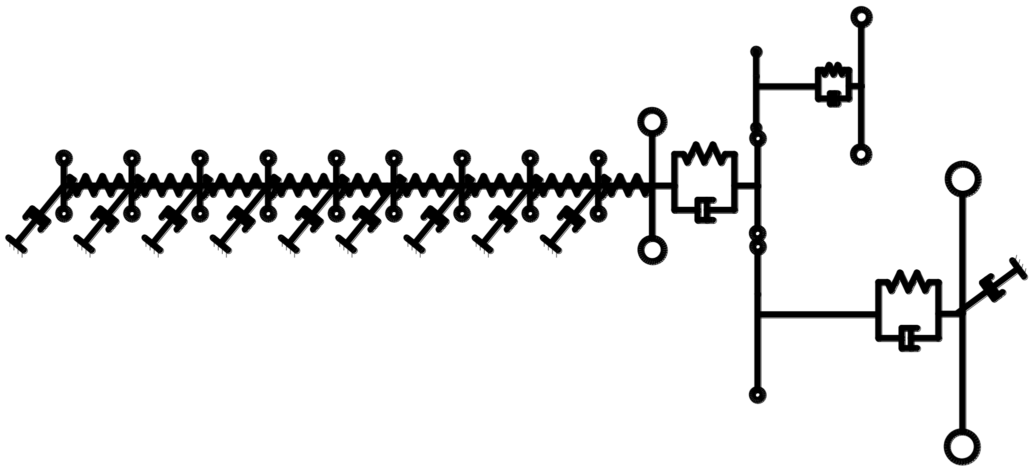






$$M_b = M_n + \sum_{i=1}^{\infty} M_{hi} . \sin(i . \varpi . t + \gamma_i)$$

2.2 Principle of replacement calculation system



A mechanical system consisting of N masses has N-1 natural frequencies, which correspond to N-1 waveforms. The decisive oscillation shapes are the lowest oscillation shapes, due to the fact that higher oscillation shapes do not usually occur in the operating speed range.

$$I.\ddot{\varphi} + k.\varphi = M_k$$

where: I – inertia matrix,
k – stiffness matrix,
M_k –vector of torque loads,
φ – vector of angles of twist,
$\ddot{\varphi}$ – vector of angular accelerations.

It is therefore sufficient for the checking calculation of the natural frequency to replace the multi-mass system with a simplified undamped two- or three-mass system.

The natural frequency of the dual mass system:

For the case of an undamped two-mass system, we can write equations of motion :

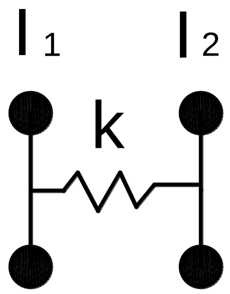
$$I_1.\ddot{\varphi}_1 + k.(\varphi_1 - \varphi_2) = M_i.\sin(i.\omega.t)$$
$$I_2.\ddot{\varphi}_2 - k.(\varphi_1 - \varphi_2) = 0$$

if:

I_1, I_2 – moments of inertia

k – stiffness

φ_1, φ_2, – twist angles



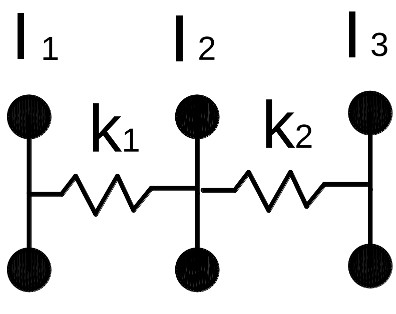
two-mass system

By solving the equations of motion, we get a relation for the natural frequency

$$\Omega_0{}^2 = k.\frac{I_1 + I_2}{I_1.I_2}$$

Natural frequency of the three-mass system:

Similar to the two-mass system, it is also possible for the three-mass undamped system to write equations of motion, the solution of which we get the circular frequency of natural oscillations equation.



three-mass system

$$\Omega_{12}{}^2 = \frac{k_1.\left(\frac{1}{I_1} + \frac{1}{I_2}\right) + k_2.\left(\frac{1}{I_2} + \frac{1}{I_3}\right)}{2} \pm \sqrt{\left(\frac{k_1.\left(\frac{1}{I_1} + \frac{1}{I_2}\right) + k_2.\left(\frac{1}{I_2} + \frac{1}{I_3}\right)}{2}\right)^2 - k_1.k_2.\frac{I_1 + I_2 + I_3}{I_1.I_2.I_3}}$$

if:

I_1, I_2, I_3 – moments of inertia

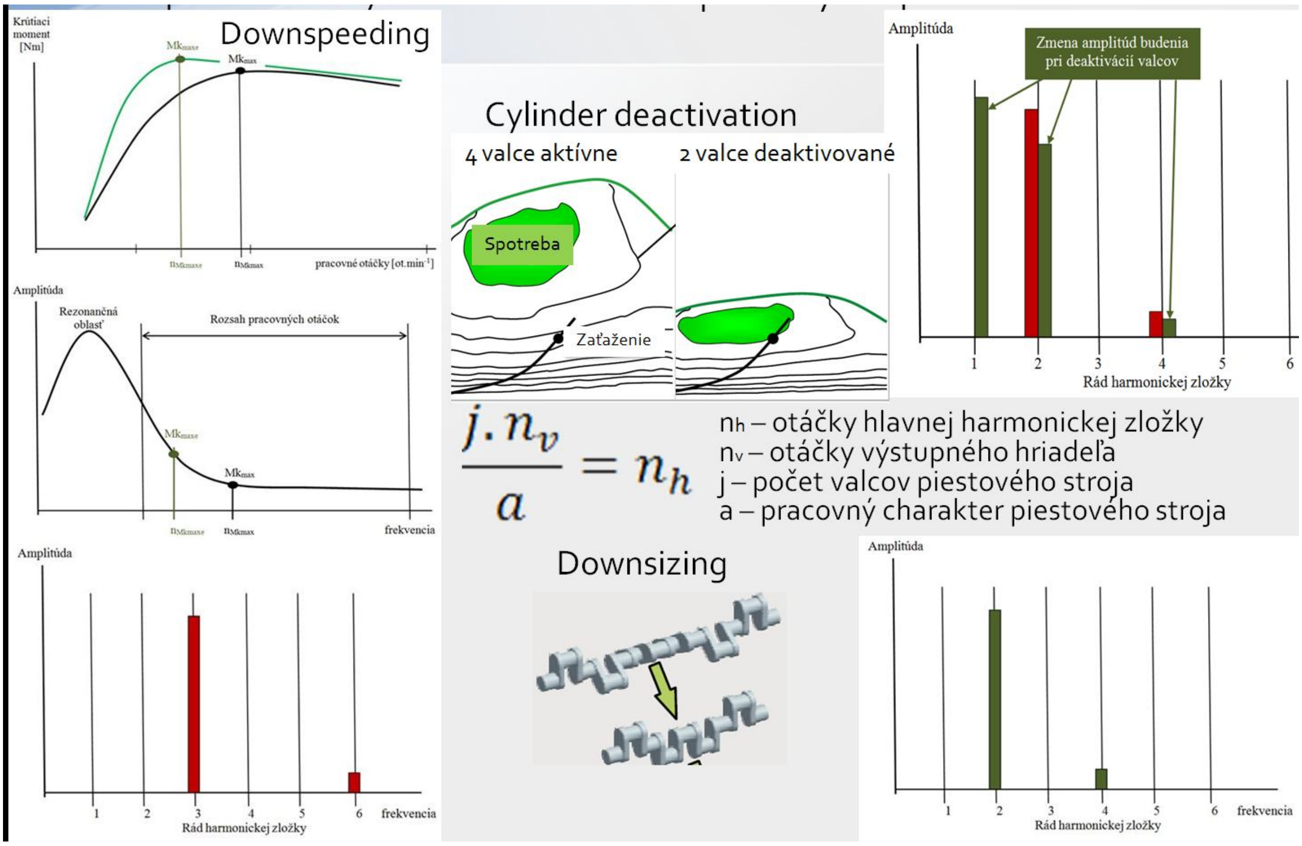
k_1, k_2 – stiffness

The most dangerous area causing disturbances is the resonance area. Resonance occurs when the natural frequency Ω_0 is equal to the excitation frequency ω of the i-th harmonic component. A suitable parameter for expressing the dynamic effects of torsional vibration on a mechanical system is the detuning factor equation

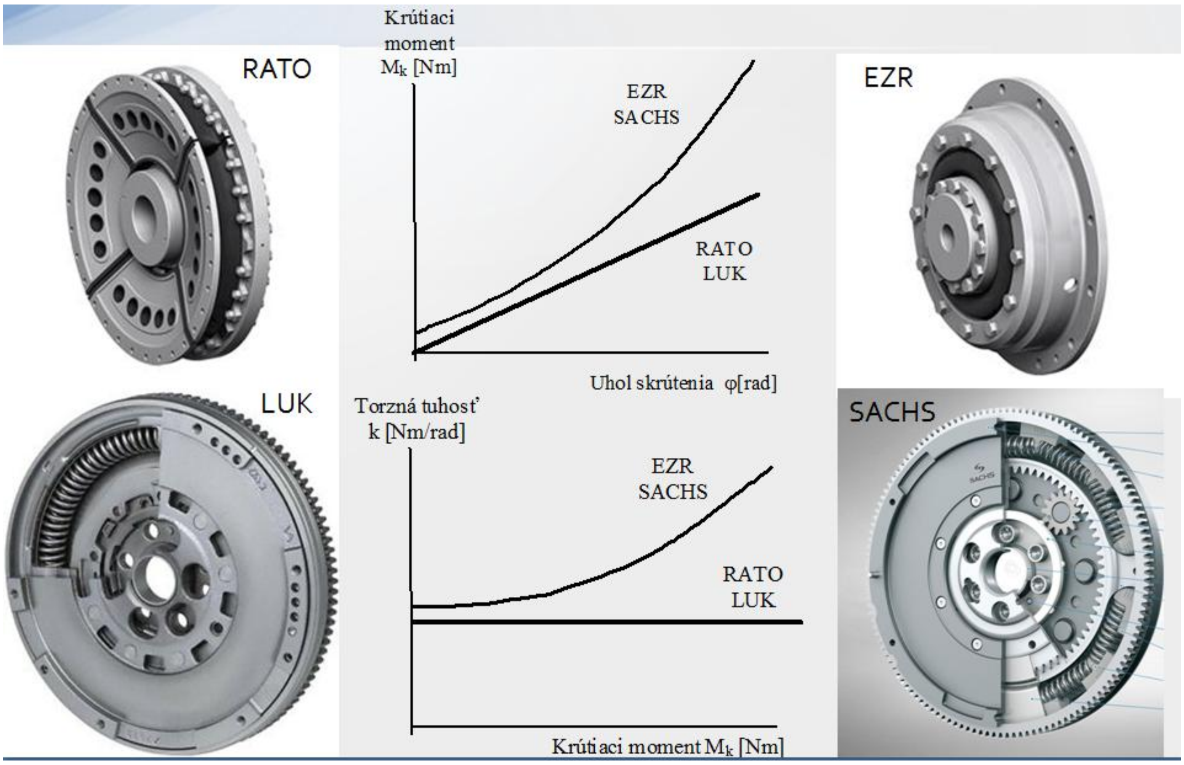
$$\eta = \frac{i.\omega}{\Omega_0}$$

In order for a mechanical system to operate without more serious resonant effects, it is desirable for the tuning factor to be in the range: $0.8 \geq \eta \geq 1.12$, for some systems operating in the over-resonant area it is desirable for the tuning factor to be $\eta \geq 1.4$.

Torsional vibration trends

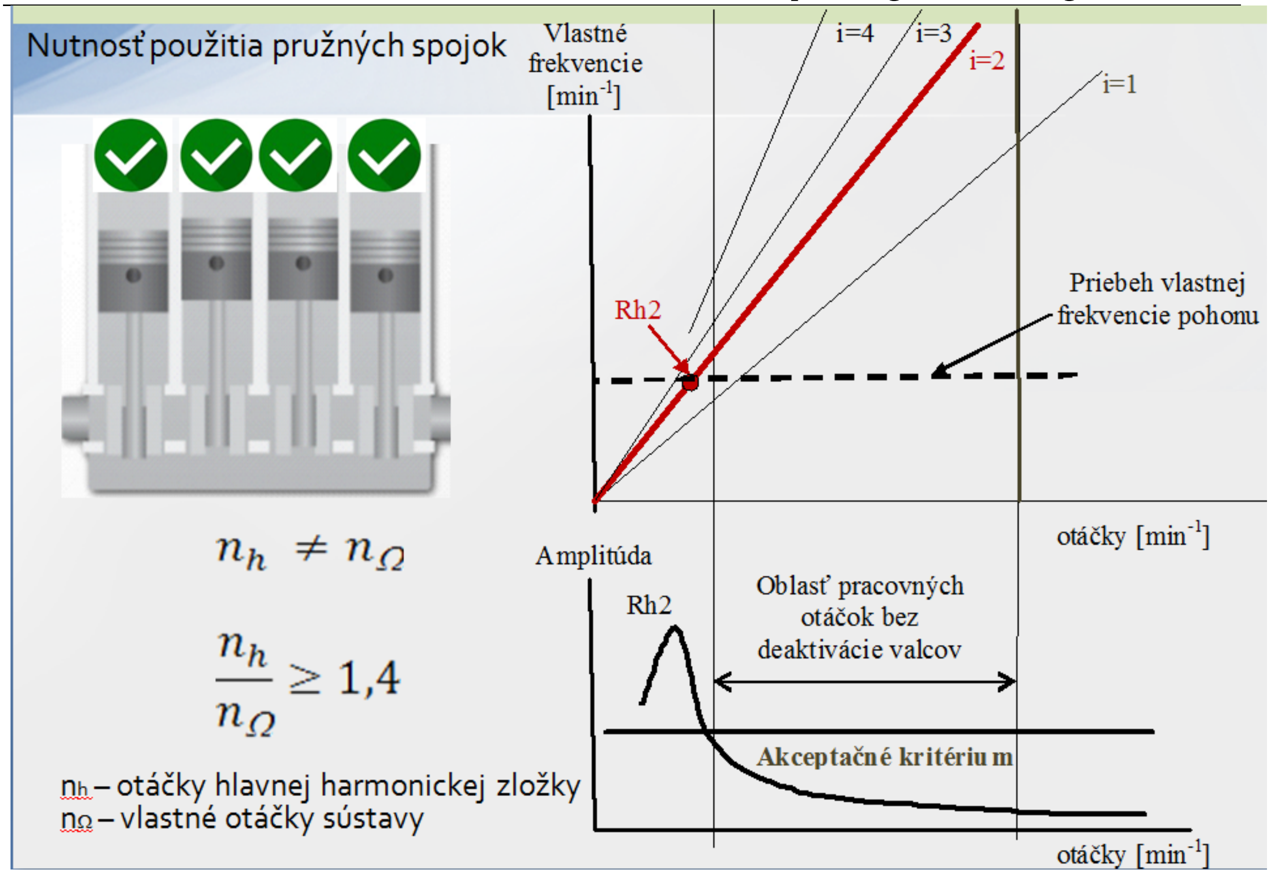


Basic Design of dual mass flywheel and flexible coupling



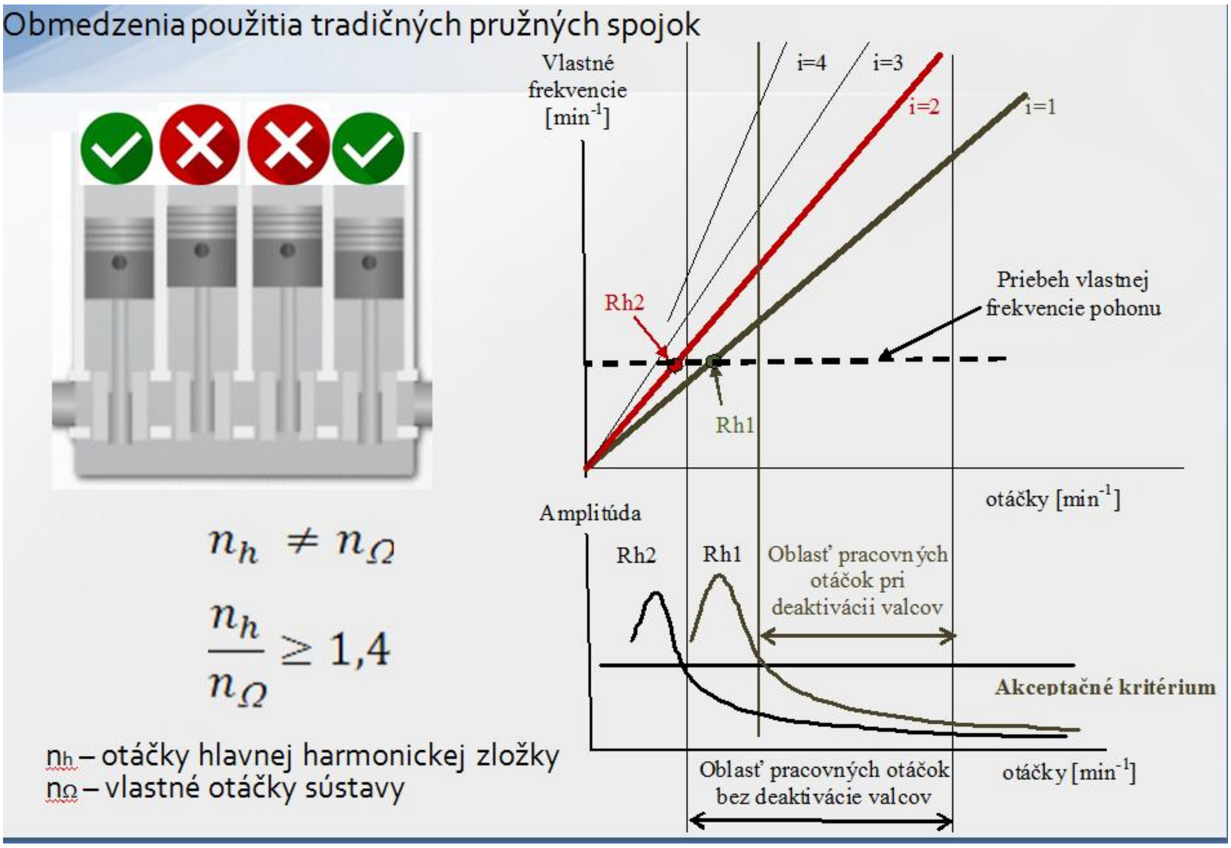
Stiffness of Dual mass flywheel:

$$k = \frac{dM_k}{d\varphi}$$



$$n_h \neq n_\Omega$$

$$\frac{n_h}{n_\Omega} \geq 1{,}4$$



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