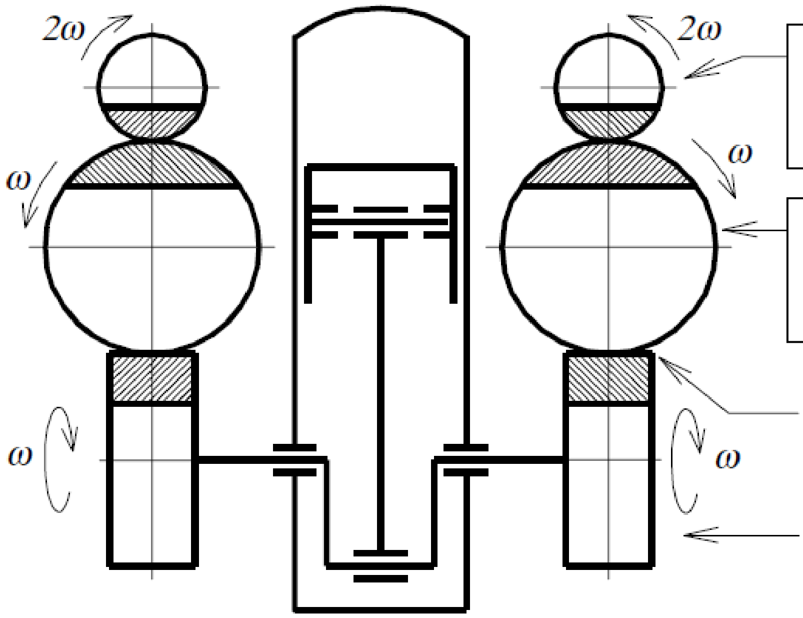
Balancing of crank mechanism

We need balancing of inertia force and moment of inertia.
Inertia force of 1st order.
Inertia force of 2nd order.
Centrifugal force.



The Balancing goals: to reduce or eliminate the effects of inertial forces and their moments, which are manifested by oscillations, noise, fatigue, etc.
Balancing is possible:
- a suitable arrangement of the crank mechanism,
- centrifugal forces can be balanced by countermass,
- inertial forces can be balanced by balancing shafts

Balancing the inertial forces of the motion parts
Balancing inertial forces of the first order:

$$F_{zpI} = m_A.r.\omega^2.cos\alpha$$
$$F_{zpI} = F_{zpI0}.cos\alpha$$

F_{zpI0} - amplitude

The goal:

$$F_{zpIv\acute{y}sl} = 0$$
$$\sum_{i=1}^{n} F_{zpIi} = 0$$

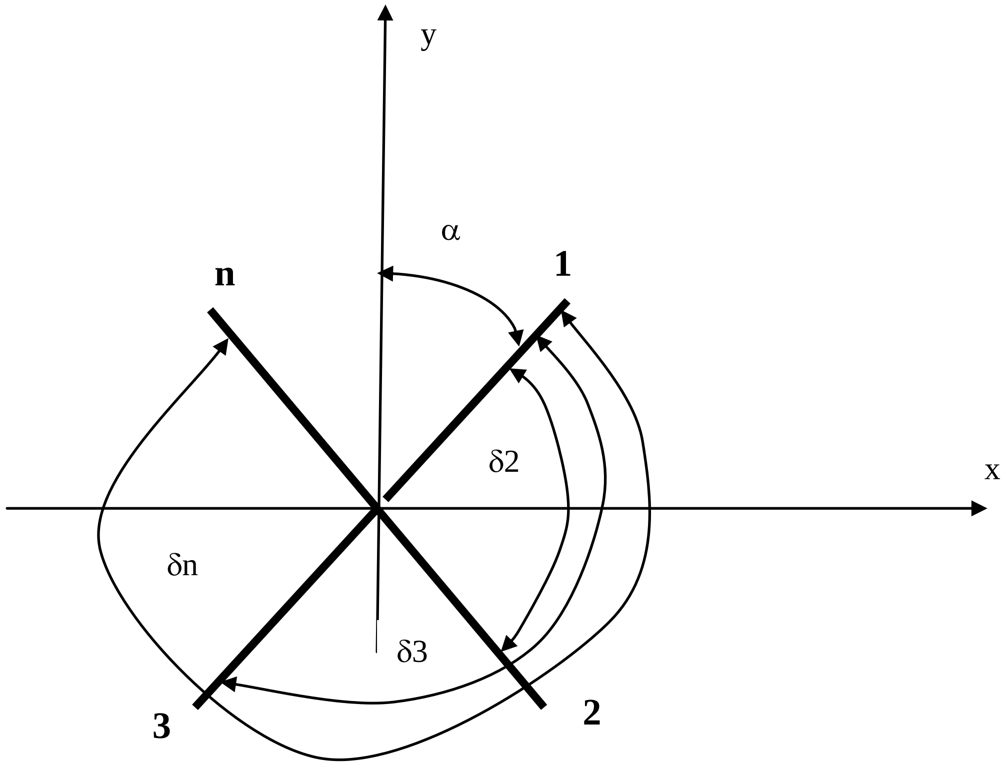
when:

i - cylinder order - position of cylinder

n - total number of crankshaft bends

α - the angle of rotation of the first bend

δi - constant angles between the first bend of crankshaft to n-th bends



Inertial Forces I.st other:

$$F_{zpI0}.cos\alpha + F_{zpI0}.\cos(\alpha+\delta_2) + F_{zpI0}.\cos(\alpha+\delta_3) + \cdots + F_{zpI0}.\cos(\alpha+\delta_n) = 0$$

substitution: $\cos(\alpha+\delta) = cos\alpha.cos\delta - sin\alpha.sin\delta$

$$F_{zpI0}.[cos\alpha.(1 + cos\delta_2 + cos\delta_3 + \cdots + cos\delta_n) - sin\alpha.(sin\delta_2 + sin\delta_3 + .. + sin\delta_n)] = 0$$

for original balancing by design of crankshaft:

$$1 + cos\delta_2 + cos\delta_3 + \cdots + cos\delta_n = 0$$
$$sin\delta_2 + sin\delta_3 + .. + sin\delta_n = 0$$

Balancing of inertial forces II.nd other:

$$F_{zpII} = m_A.r.\omega^2.\lambda.cos2\alpha$$
$$F_{zpII} = F_{zpII0}.cos2\alpha$$

F_{zpII0} - amplitude

The goal:

$$F_{zpIIv\acute{y}sl} = 0$$
$$\sum_{i=1}^{n} F_{zpIIi} = 0$$

Inertial Forces II.nd other:

$$F_{zpII0}.[cos2\alpha + cos2(\alpha+\delta_2) + cos2(\alpha+\delta_3) + \cdots + cos2(\alpha+\delta_n)\ ] = 0$$

for original balancing by design of crankshaft:

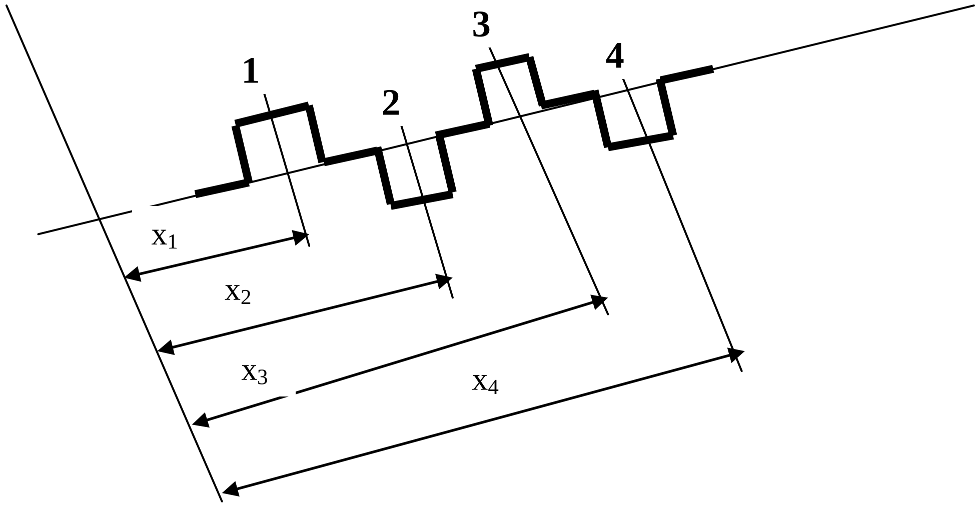
$$1 + cos2\delta_2 + cos2\delta_3 + \cdots + cos2\delta_n = 0$$
$$sin2\delta_2 + sin2\delta_3 + .. + sin2\delta_n = 0$$

Balancing Inertia force of rotating parts:

The goal:

$$F_{zprv\acute{y}sl} = 0$$
$$\sum_{i=1}^{n} F_{zpri} = 0$$

Moment balancing of an internal combustion engine



Balancing moments from motion inertial forces:

$$M_{zp} = m_A.r.\omega^2.\{x_1.[cos\alpha + \lambda.cos2\alpha] + \cdots$$
$$+ x_n.[(cos\alpha + \delta_n) + \lambda.cos2(\alpha + \delta_n)]\}$$

after:

Balancing moments I.st other:

$$x_1 + x_2.cos\delta_2 + x_3.cos\delta_3 + \cdots + x_n.cos\delta_n = 0$$
$$x_2.sin\delta_2 + x_3.sin\delta_3 + \cdots + x_n.sin\delta_n = 0$$

Balancing moments II.nd other:

$$x_1 + x_2.cos2\delta_2 + x_3.cos2\delta_3 + \cdots + x_n.cos2\delta_n = 0$$
$$x_2.sin2\delta_2 + x_3.sin2\delta_3 + \cdots + x_n.sin2\delta_n = 0$$

Balancing moments of rotating parts:

$$x_1 + x_2.cos\delta_2 + x_3.cos\delta_3 + \cdots + x_n.cos\delta_n = 0$$
$$x_2.sin\delta_2 + x_3.sin\delta_3 + \cdots + x_n.sin\delta_n = 0$$

Finally moment from motion and rotating parts:

$$\overline{M_{výsl}} = \overline{M_{zp}} + \overline{M_{zr}}$$

If the resultant moment is not "0", then the resultant moment must be balanced against the countermass and balancing shafts. The magnitude of the balancing moment will be:

$$\overline{-M_{výsl}} = \overline{M_{vyvaž}}$$

Flywheel

During the working cycle in the cylinder, there is an acceleration of the shaft when there is an excess of energy and it slows down when there is a lack of energy. This difference can be expressed by the degree of uneven running.

$$\delta_s = \frac{\omega_{max} - \omega_{min}}{\omega_s}$$

or:

$$\delta_s = \frac{n_{max} - n_{min}}{n_s}$$

if:

$$\omega_s = \frac{\omega_{max} + \omega_{min}}{2}$$

or:

$$n_s = \frac{n_{max} + n_{min}}{2}$$

n_{max}, n_{min} - maximum and minimum speed during the work cycle!

Recommended values for uneven engine running:
automobile engines δs = 1/180 to 1/300
aircraft engines δs = 1/1000

Energy change during uneven motion:

$$\Delta E_k = \frac{1}{2}.I.\left(\omega_{max}^2 - \omega_{min}^2\right) = \frac{1}{2}.(I_z + I_o).\left(\omega_{max}^2 - \omega_{min}^2\right)$$

I_z - inertia mass moment of the flywheel
I_o - inertia mass moment other rotating parts

after:

$$\left(\omega_{max}^2 - \omega_{min}^2\right) = \frac{2.\Delta E_k}{(I_z + I_o)}$$

- uneven running can be influenced by ΔE_k or $I_z + I_o$

The aim is to reduce the value of uneven running. A reduction of ΔE_k can be achieved by increasing the number by the engine cylinder. The change with $I_z + I_o$ means an increase in the mass moment of inertia of the engine (larger flywheel) - which has an adverse effect on the acceleration of the mass of the engine.

Define of inertia mass of flywheel:

$$\Delta E_k = \frac{1}{2}.(I_o + I_z).\left(\omega_{max}^2 - \omega_{min}^2\right)$$
$$= (I_o + I_z).\frac{\left(\omega_{max} - \omega_{min}\right).\left(\omega_{max} + \omega_{min}\right)}{2}$$
$$= (I_o + I_z).\delta_s.\omega_s^2$$

$$I_z = \frac{\Delta E_k}{\delta_s.\omega_s^2} - I_o$$

power of engine:

$$\Delta A = \int (M_k - M_{hn}).d\alpha$$

M_{hn} - torque for driven devices

Then:

$$I_z = \frac{\Delta A}{\delta_s.\omega_s^2} - I_o$$

Other factors taking into account the flywheel design:
- transients during regulation,
- time required to stop the engine,
- rigidity of the connection between the engine and the driven machine,
- special tasks.