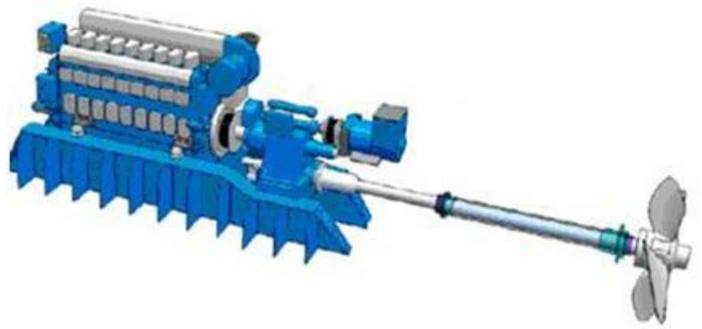
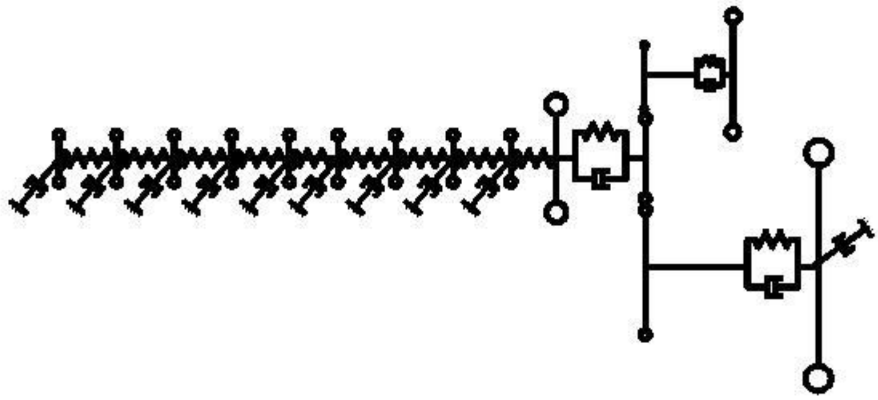
Design of shaft couplings for mechanical systems

To ensure the correct and functional operation of the mechanical system (drive), it is necessary to properly analyze all operating conditions. The main parts of the drives are the driving machine (drive) - gearbox - and the driven machine (appliance). The individual drive parts are connected by shafts on which are clutches, brakes and other machine parts.





Mass moment of inertia of the disk:

$$I = \frac{\pi}{32} . d^4 . h . \rho$$

if:
d- diameter of disc
h- width of disc
ρ- density of disc

The law of conservation of kinetic energy must apply when creating a replacement system $E_{kp} = E_{kn}$.
Kinetic energy of original system:

$$E_{kp} = \frac{1}{2} . I_p . \omega_p^2$$

Kinetic energy of replacement system:

$$E_{kn} = \frac{1}{2} . I_n . \omega_n^2$$

Substitution:

$$\frac{1}{2}.I_n.\omega_n^2 = \frac{1}{2}.I_p.\omega_p^2$$

Mass moment of inertia of the replacement disk:

$$I_n = I_p.\left(\frac{\omega_p}{\omega_n}\right)^2$$

The material discs that are on a common shaft have the same angular velocity ω. In this case, the resulting moment can be determined as the sum of the partial moments of inertia:

$$I_{vys} = \sum I_{ni}$$

After reducing the masses on the shaft on which the shaft coupling is located, it is possible to proceed to the design of the coupling. The design of the coupling itself can then be done using three methods:

1. Determination of the clutch size according to the operating factor K.
2. Determination of the size of the clutch from the spare drive system.
3. Determination of clutch size from dynamic drive calculation.

1. Determination of the clutch size according to the operating factor K

This is the fastest but least accurate calculation method for the design of a shaft coupling.

The calculated torque of the clutch is determined:

$$M_{kv} = K.M_k$$

if:
K – operating factor
M_k – torque
P- power
ω – angular velocity
n – revolutions

$$M_k = \frac{P}{\omega} = \frac{P}{2.\pi.n}$$

The "K" factor is chosen according to the type of driving and driven machine, in addition the number of clutch engagements and other conditions must be taken into account.

$$K = K_1.K_2.K_3.K_4$$

if:
K_1 – type of driving machine (0,33-1,25)
K_2 – type of driven machine (1 – 2,5)
K_3 – specific wear factor - for turn on and turn off couplings (0,1-0,43)
K_4 – coefficient of amount of slip of friction surfaces (0,2-2)

If we have a reduced mechanical system on the clutch shaft, and the mass moments of inertia "I" are known which need to be accelerated "ε".

The following cases may occur:

1.) Fixed clutch load during start-up of the "M_{kr}" drive and input torque M_{k1max} and $M_{k2} = 0$. The moment "M_{k1max}" accelerates both masses and then for the acceleration of rotating masses the impact moment will be:

$$M_{kr} = I_2 . \varepsilon_{max} = M_{k1max} \frac{I_2}{I_1 + I_2} = K . M_{kv}$$

2.) Fixed coupling load at periodic variable torque if input torque M_{k1} and constant torque $M_{k2} = M_{kv} = const.$

Excess driving torque (M_{k1max} - M_{k2}) accelerates both masses and then applies the acceleration of rotating masses:

$$M_{kr} = I_2 . \varepsilon_{max} = (M_{k1max} - M_{kv}) . \frac{I_2}{I_1 + I_2} = K . M_{kv}$$

Maximal load of coupling:

$$M_{kvmax} = (1 + K) . M_{kv}$$

3.) Fixed coupling load at periodic variable torque if input torque M_{k2} and constant torque $M_{k1} = M_{kv} = const.$

Then it applies to the acceleration of rotating masses:

$$M_{kr} = I_2 . \varepsilon_{max} = (M_{k2max} - M_{kv}) . \frac{I_2}{I_1 + I_2} = K . M_{kv}$$

Maximal load of coupling:

$$M_{kvmax} = M_{kv} + M_{kr} = (1 + K) . M_{kv}$$

4.) Fixed clutch load at periodically variable moments if M_{k2} and M_{k1}.

Excess driving torque (M_{k1max} - M_{k2max}) accelerates both masses and then applies the acceleration of rotating masses:

$$M_{kr} = I_2 . \varepsilon_{max} = (M_{k1max} - M_{k2max}) . \frac{I_2}{I_1 + I_2} = K . M_{kv}$$

Maximal load of coupling:

$$M_{kvmax} = M_{kv} + M_{kr} = (1 + K) . M_{kv}$$

2. Determination of the size of the clutch from the spare drive system.

When the replacement system is ingested, we replace the real rotating masses with spare disks and create an n-mass dynamic system. We reduce this n-mass system to a simpler two- or three-mass system. We realize the reduction on the shaft on which the shaft coupling we want to design is located.

When replacing masses and reducing the n-mass system to a simpler computational system, we proceed in accordance with the law of conservation of kinetic energy as stated in the introduction to this block.

This method of coupling design has the following limiting conditions:

- Reduction of materials on the shaft on which the coupling is located.
- Neglecting the impact of misalignment of the steering.
- Neglecting the effect of shaft torsion and flexible coupling.
- The flexible coupling has a linear characteristic.
- In the case of friction surfaces, the friction torque is uniform from the beginning to the end of the slip.
- Constant acceleration in case of friction surfaces.
- Neglecting the deceleration of the driven side at start-up, as opposed to the kneading side.
- Neglect of angular play in the case of a friction clutch.

By accepting the above boundary conditions, we can analyze the following operating conditions:

1. The starting torque of the drive part is constant and the torque of the driven part is constant

<u>For mechanically uncontrolled couplings</u>:

Ak φ =0

$$M_{kv} = M_{k2} + M_{kr}.\frac{I_2}{I_1 + I_2}$$
$$M_{kr} = M_{k1max} - M_{k2}$$

Ak φ >0

$$M_{kv} = M_{k2} + M_{kr}.\frac{I_2}{I_1 + I_2} + \sqrt{2.M_{1s}.\varphi.k.\frac{I_2}{I_1 + I_2}}$$
$$M_{kv} = M_{k1max} - M_{k2}$$
$$M_{1s} = \frac{1}{t_t}.\int_0^{t_t} M_{k1}.dt$$

For electro motors $M_{1s} \leq 2.M_k$

k- torsional stiffness of coupling

t_t- the time from engaging the drive to engaging the functional parts of the clutch that correspond to the angular play φ.

Applies to friction clutch:
At maximum temperature load if $z \leq z_t$:

$$M_{kv} = M_{k2} + \frac{10{,}68.\,I_2.\,n_r^2}{10^6.\,t_{max}}$$

if $z_{max} > z > z_t$:

$$M_{kv} = M_{k2} + \frac{10{,}68.\,I_2.\,n_r^2.\,z}{10^6.\,t_{max}.\,z_t}$$

z- number of switchings per hour
z_t- permissible number of clamps per hour due to the thermal load of the coupling
z_{max} – the maximum achievable number of switchings per hour
n_r- the speed at which the clutch can slip permanently
t_{max} – the maximum permissible slip time of the clutch (s)

$$n_r \leq \frac{t_{max}.\,z_t}{7{,}2}$$

At a given slip time:

$$M_{kv} = M_{k2} + \frac{10{,}68.\,I_2.\,n_r}{10^3.\,t_{max}}$$

From conditional $z \leq z_t$:

$$t \leq t_{max}.\frac{1000}{n_r}$$

From conditional $z_t < z < z_{max}$:

$$t \leq t_{max}.\frac{1000.\,z_t}{n_r.\,z}$$

2. The torque of the drive part is periodically variable and the torque of the driven part is constant

For mechanically uncontrolled couplings:

$$M_{kv} = M_{k2} + M_{kr}.\frac{I_2}{I_1 + I_2}$$
$$M_{kr} = M_{k1max} - M_{k2}$$

if $\varphi > 0$

$$M_{k1} > -M_{k2}.\frac{I_1}{I_2}$$

Applies to friction clutch:

$$M_{kv} = M_{k2} + M_{kr}.\frac{I_2}{I_1 + I_2}$$
$$M_{kr} = M_{k1max} - M_{k2}$$

3. The torque of the drive part is constant and the torque of the driven part is variable

For mechanically uncontrolled couplings:

$$M_{kv} = M_{k1} + M_{kr}.\frac{I_1}{I_1 + I_2}$$
$$M_{kr} = M_{k2max} - M_{k1}$$

if φ >0

$$M_{k2} > -M_{k1}.\frac{I_2}{I_1}$$

Applies to friction clutch:

$$M_{kv} = M_{k1} + M_{kr}.\frac{I_1}{I_1 + I_2}$$
$$M_{kr} = M_{k2max} - M_{k1}$$

4. The torque of the drive part and the torque of the driven part are variable

Applies to friction clutch and For mechanically uncontrolled couplings:
Larger moment decides M_{kv}

$$M_{kv} = M_{k2} + M_{kr}.\frac{I_2}{I_1 + I_2}$$
$$M_{kr} = M_{k1max} - M_{k2max}$$
$$M_{kv} = M_{k1} + M_{kr}.\frac{I_1}{I_1 + I_2}$$
$$M_{kr} = M_{k2max} - M_{k1max}$$

if φ >0

$$M_{k1} > -M_{k2min}.\frac{I_1}{I_2}$$

$$M_{k2} > -M_{k2min}.\frac{I_2}{I_1}$$

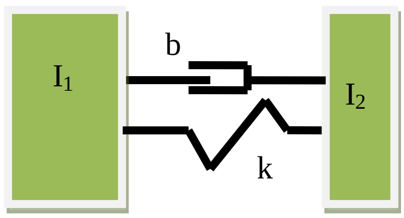
3. Determination of clutch size from dynamic drive calculation.

This method is the most accurate way to design a shaft coupling. It is necessary to know the detailed parameters of all parts of the drive, their material moments of inertia, stiffness, etc. Above all, it is necessary to know the type of drive and driven equipment, its performance characteristics, speed, etc. It is necessary for the coupling to know the torsional rigidity of the coupling, its damping and other parameters.

This method is also based on the reduction of the n-mass system to a simpler two- or three-mass system. We realize the reduction on the shaft on which the shaft coupling we want to design is located. We will achieve a more accurate solution by reducing it to a three-mass system, but the calculation is more demanding. Therefore, a dual-mass computing system is used more in practice.

Two mass damped system:

For the two-mass damped system in Fig. We can write equations of motion in the form:



$$I_1.\ddot{\varphi}_1 + b.(\dot{\varphi_1} - \dot{\varphi_2}) + \ k.(\varphi_1 - \varphi_2) = M_{ki}.\sin\,(i.\omega.t)$$
$$I_2.\ddot{\varphi}_2 + +b.(\dot{\varphi_2} - \dot{\varphi_1}) + \ k.(\varphi_2 - \varphi_1) = 0$$

By solving the differential equations we get the natural frequency of the system in the form:

$$\Omega = \sqrt{k.(\frac{1}{I_1} + \frac{1}{I_2})}$$

Resonance will occur if it is valid:

$$\Omega = \omega.i$$

The value of the dynamic torque component:

$$\mathrm{M_{kd}} = \sum M_{ki}.\frac{I_2}{I_1 + I_2}.\frac{\sqrt{1 + \left(\frac{i.\omega}{\Omega}\right)^2.\left(\frac{2.\xi}{\Omega}\right)^2}}{\sqrt{\left[1 - \left(\frac{i.\omega}{\Omega}\right)^2\right]^2 + \left(\frac{i.\omega}{\Omega}\right)^2.\left(\frac{2.\xi}{\Omega}\right)^2}}$$

if:

$$2.\xi = b.\left(\frac{1}{I_1} + \frac{1}{I_2}\right)$$

Clutch calculation torque:

$$M_{kv} = M_{kst} + M_{kd}$$

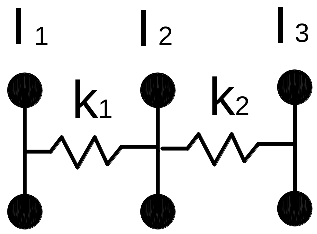
Dynamic moment for a two-mass undamped system:

$$M_{kd} = \frac{M_{ki}.I_2}{I_1 + I_2}.\frac{1}{\left[1 - \left(\frac{i.\omega}{\Omega}\right)\right]}$$

if
M_{ki} – moment amplitude of the harmonic component of the i-th order

Dynamic moment for a three-mass undamped system:

Similar to the two-mass system, it is also possible for the three-mass undamped system Fig., To write equations of motion, the solution of which we get the frequency of natural oscillations equation.



$$\Omega_{1,2}^{\ 2} = \frac{k_1.\left(\frac{1}{I_1} + \frac{1}{I_2}\right) + k_2.\left(\frac{1}{I_2} + \frac{1}{I_3}\right)}{2} \pm \sqrt{\left(\frac{k_1.\left(\frac{1}{I_1} + \frac{1}{I_2}\right) + k_2.\left(\frac{1}{I_2} + \frac{1}{I_3}\right)}{2}\right)^2 - k_1.k_2.\frac{I_1 + I_2 + I_3}{I_1.I_2.I_3}}$$

if:
I_1, I_2, I_3 – mass moments of inertia
k_1, k_2 – torsional stiffness

The most dangerous area causing disturbances is the resonance area. Resonance occurs when the natural frequency Ω_0 is equal to the excitation frequency ω from the i-th harmonic component.