DEPARTMENT OF APPLIED MATHEMATICS AND INFORMATICS

FACULTY OF MECHANICAL ENGENEERING

TECHNICAL UNIVERSITY OF KOŠICE

BASIC MATH FORMULAS

2025

BASIC ALGEBRA FORMULAS

1) $(a+b)^2 = a^2 + 2ab + b^2$

2) $(a-b)^2 = a^2 - 2ab + b^2$

3) $a^2 - b^2 = (a-b)(a+b)$

4) $(a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$

5) $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

6) $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

EXPONENT RULES

1) $a^m \cdot a^n = a^{m+n}$

2) $a^m : a^n = a^{m-n}$

3) $(a^m)^n = a^{m \cdot n}$

4) $a^0 = 1$

5) $a^{-n} = \frac{1}{a^n}$

6) $(a \cdot b)^m = a^m \cdot b^m$

7) $\sqrt[n]{a} = a^{\frac{1}{n}}$

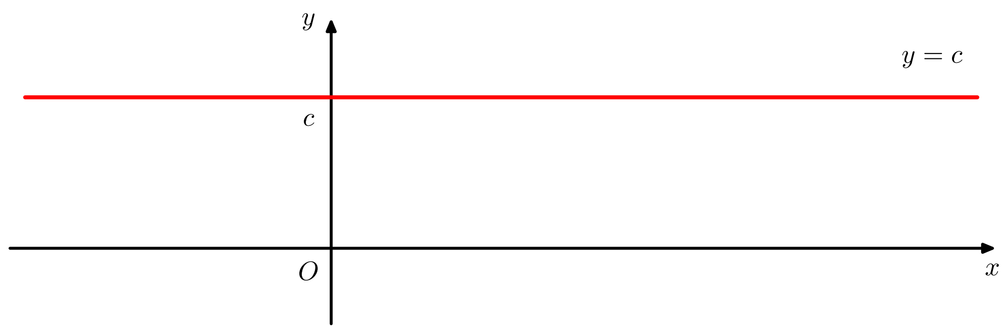
8) $\sqrt[n]{a^m} = a^{\frac{m}{n}}$

ELEMENTARY FUNCTIONS

Constant function – $\boxed{f:\ y = c,\ c \in \mathbb{R}}$

$\mathcal{D}(f) = \mathbb{R},\ \mathcal{R}(f) = \{c\}.$

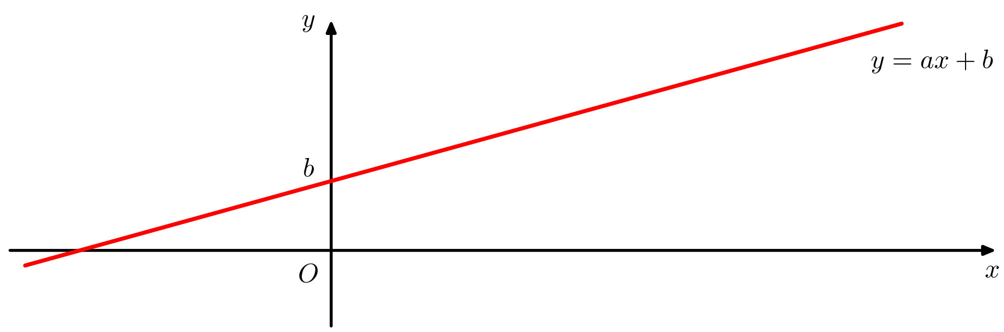
The graph is a line parallel to the x-axis.



Linear function – $\boxed{f:\ y = ax + b,\ a, b \in \mathbb{R},\ a \neq 0}$

$\mathcal{D}(f) = \mathbb{R},\ \mathcal{R}(f) = \mathbb{R}.$

The graph is a line, where a is the slope and b is the y-intercept.



Quadratic function – $\boxed{f:\ y = ax^2 + bx + c,\ a, b, c \in \mathbb{R},\ a \neq 0}$

The graph is a parabola with axis of symmetry parallel to the y-axes.
1) $a > 0$:

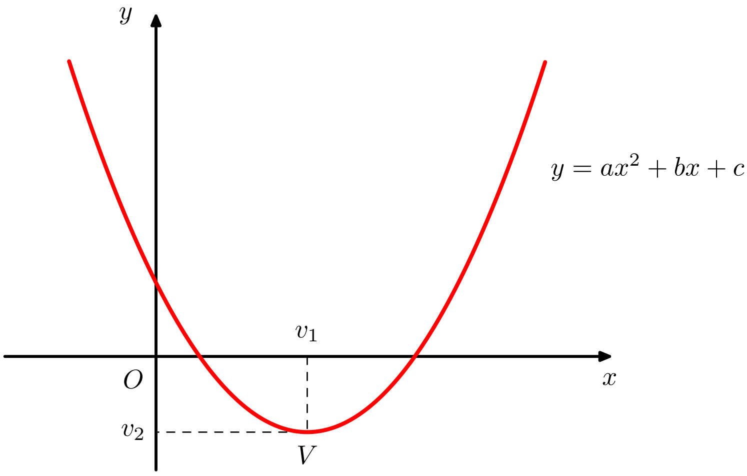
$\mathcal{D}(f) = \mathbb{R},\ \mathcal{R}(f) = \langle v_2; \infty),$

even for $b = 0$,

bounded from below,

decreasing, one-to-one when $x \in (-\infty; v_1\rangle$,

increasing, one-to-one when $x \in \langle v_1; \infty)$.



2) $a < 0$:

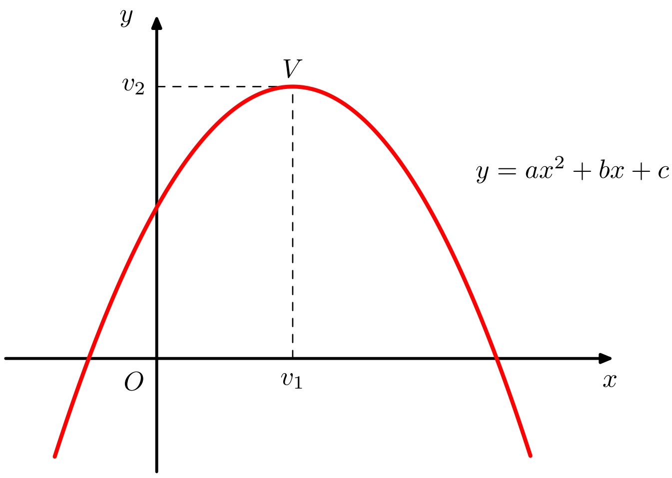
$\mathcal{D}(f) = \mathbb{R},\ \mathcal{R}(f) = (-\infty; v_2\rangle,$

even for $b = 0$,

bounded from above,

increasing, one-to-one when $x \in (-\infty; v_1\rangle$,

decreasing, one-to-one when $x \in \langle v_1; \infty)$.



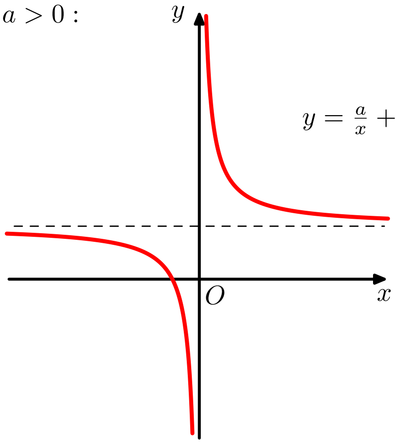
Let x_1 and x_2 be the roots of the quadratic equation $ax^2 + bx + c = 0$. Then the quadratic function $y = ax^2 + bx + c$ can be written in the form $y = a(x - x_1)(x - x_2)$.

Rational function – $\boxed{f:\ y = \dfrac{a}{x} + b,\ a, b \in \mathbb{R},\ a \neq 0}$

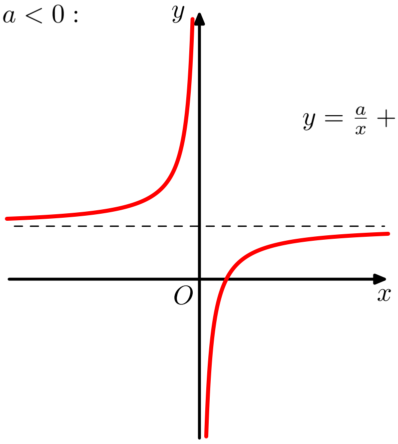
$\mathcal{D}(f) = (-\infty; 0) \cup (0; \infty),\ \mathcal{R}(f) = (-\infty; b) \cup (b; \infty).$

The graph is a rectangular hyperbola.

$a > 0:$



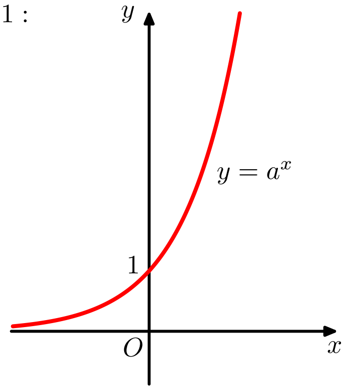
$a < 0:$



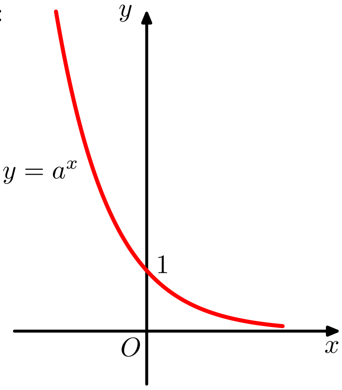
Exponential function – $\boxed{f:\ y = a^x,\ a > 0,\ a \neq 1}$

$\mathcal{D}(f) = \mathbb{R},\ \mathcal{R}(f) = (0; \infty).$

$a > 1:$



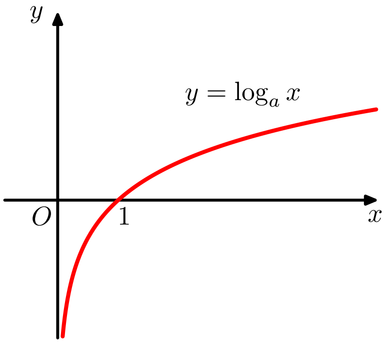
$0 < a < 1:$



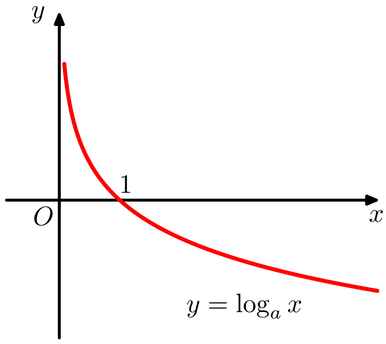
Logarithmic function – $\boxed{f:\ y = \log_a x,\ a > 0,\ a \neq 1}$

$\mathcal{D}(f) = (0; \infty),\ \mathcal{R}(f) = \mathbb{R}.$

$a > 1:$



$0 < a < 1:$



$\log x = \log_{10} x$ is called the decimal or common logarithm,

$\ln x = \log_e x$ is called the natural logarithm, (where $e = 2.718\ldots$ is Euler's number).

Properties of logarithms:

$$\log_a xy = \log_a x + \log_a y \qquad \log_a 1 = 0$$

$$\log_a \frac{x}{y} = \log_a x - \log_a y \qquad \log_a a = 1$$

$$\log_a x^n = n \cdot \log_a x \qquad \log_a x = \frac{\log_y x}{\log_y a}$$

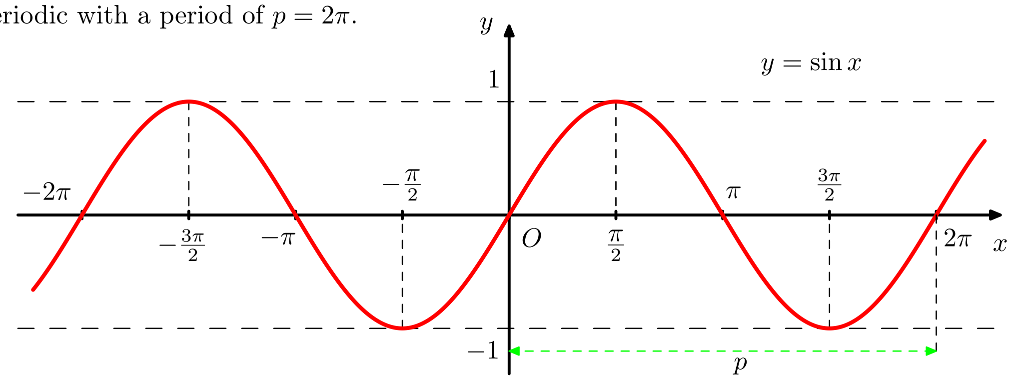
Trigonometric functions

$\boxed{f:\ y = \sin x}$ $\quad \mathcal{D}(f) = \mathbb{R},\ \mathcal{R}(f) = \langle -1; 1 \rangle,$

odd, thus $\quad \sin(-x) = -\sin x,$

bounded,

periodic with a period of $p = 2\pi$.

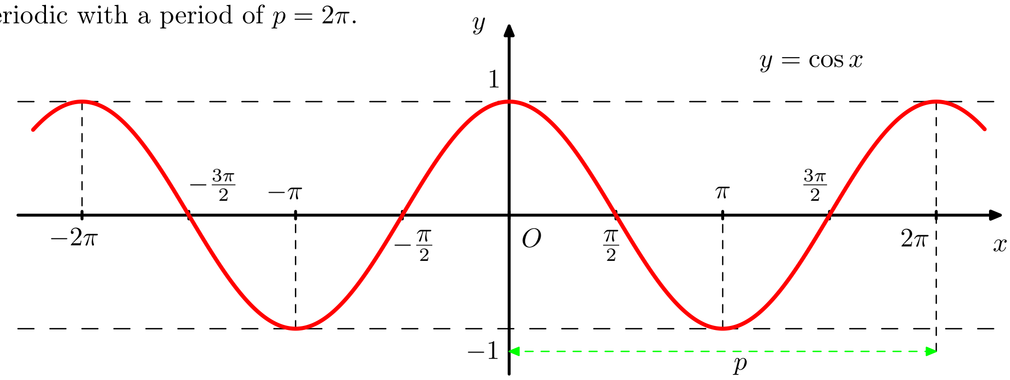


$\boxed{f:\ y = \cos x}$ $\quad \mathcal{D}(f) = \mathbb{R},\ \mathcal{R}(f) = \langle -1; 1 \rangle,$

even, thus $\quad \cos(-x) = \cos x,$

bounded,

periodic with a period of $p = 2\pi$.



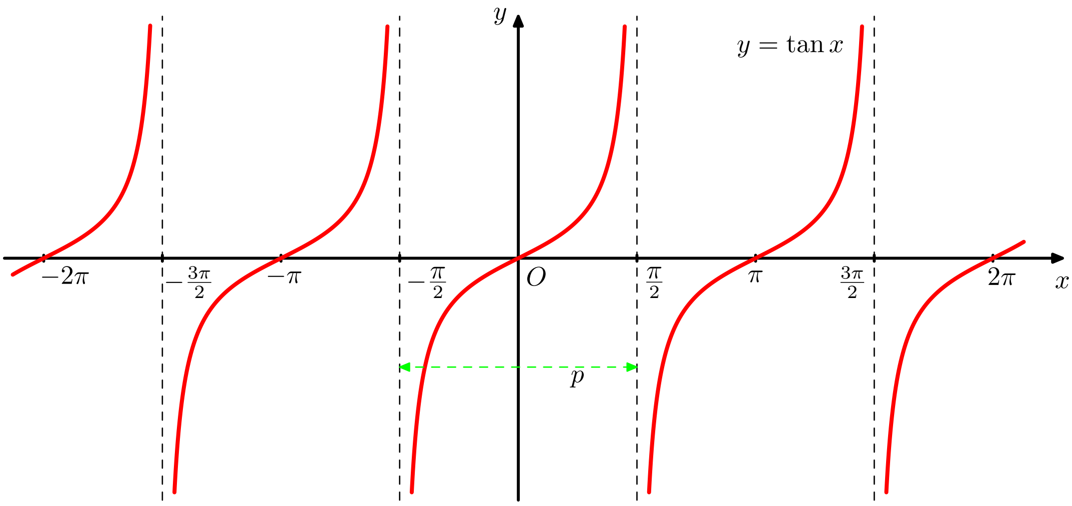
$\boxed{f:\ y = \tan x}$ $\quad \mathcal{D}(f) = \mathbb{R} - \{(2k+1)\cdot \frac{\pi}{2};\ k \in \mathbb{Z}\} = \bigcup_{k\in\mathbb{Z}} (-\frac{\pi}{2} + k\pi;\ \frac{\pi}{2} + k\pi),\ \mathcal{R}(f) = \mathbb{R},$

odd, thus $\quad \tan(-x) = -\tan x,$

increasing on every interval $I \subset \mathcal{D}(f)$,

unbounded,

periodic with a period of $p = \pi$.



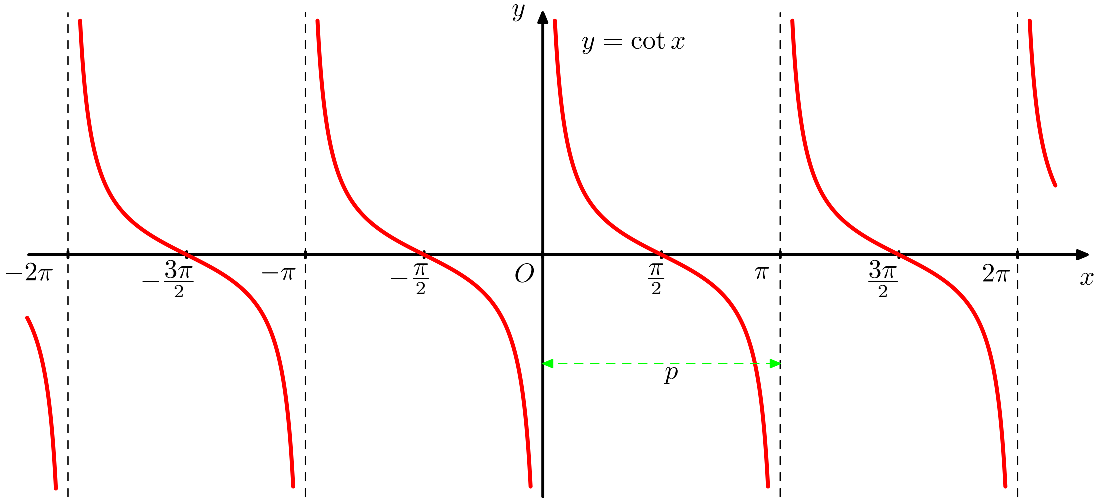
$\boxed{f:\ y=\cot x}$ $\mathcal{D}(f)=\mathbb{R}-\{k\pi; k\in\mathbb{Z}\}=\bigcup_{k\in\mathbb{Z}}(k\pi;(k+1)\pi)$, $\mathcal{R}(f)=\mathbb{R}$,

odd, thus $\cot(-x)=-\cot x$,

decreasing on every interval $I\subset\mathcal{D}(f)$,

unbounded,

periodic with a period of $p=\pi$.



Signs of trigonometric functions

quadrant	$\sin x$	$\cos x$	$\tan x$	$\cot x$
I.	+	+	+	+
II.	+	−	−	−
III.	−	−	+	+
IV.	−	+	−	−

$\sin(90°-x)=\cos x$

$\cos(90°-x)=\sin x$

$\sin(90°+x)=\cos x$

$\cos(90°+x)=-\sin x$

Values of the trigonometric functions

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan x$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	*
$\cot x$	*	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

$1\,\text{radian}=\frac{180°}{\pi}\doteq 57°\,17'\,45''$

$1°=\frac{\pi}{180}\doteq 0{,}0175$

1) $\boldsymbol{\sin^2 x + \cos^2 x = 1}$

2) $\tan x = \dfrac{\sin x}{\cos x} = \dfrac{1}{\cot x}$

3) $\cot x = \dfrac{\cos x}{\sin x} = \dfrac{1}{\tan x}$

4) $\boldsymbol{\sin 2x = 2\sin x \cos x}$

5) $\boldsymbol{\cos 2x = \cos^2 x - \sin^2 x}$

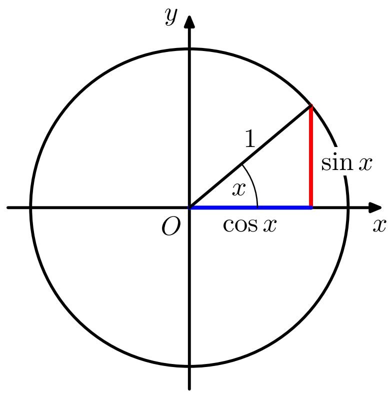
6) $\sin^2 \dfrac{x}{2} = \dfrac{1-\cos x}{2}$

7) $\cos^2 \dfrac{x}{2} = \dfrac{1+\cos x}{2}$

8) $\sin(x \pm y) = \sin x \cdot \cos y \pm \cos x \cdot \sin y$

9) $\cos(x \pm y) = \cos x \cdot \cos y \mp \sin x \cdot \sin y$

Unit circle:



QUADRATIC EQUATIONS

The quadratic equation in the standard form is $\boxed{ax^2 + bx + c = 0,\ \ a \neq 0,\ \ a, b, c \in \mathbb{R}}$. The roots x_1, x_2 of a quadratic equation are given by

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a},$$

where the expression $D = b^2 - 4ac$ is called the discriminant.
If $D > 0$ then the quadratic function has two distinct real roots.
If $D = 0$ then the quadratic function has one repeated real root

$$x_1 = x_2 = -\frac{b}{2a}.$$

If $D < 0$ then the quadratic function has two complex roots

$$x_{1,2} = \frac{-b \pm i \cdot \sqrt{|D|}}{2a},$$

where i is the imaginary unit.

If x_1, x_2 are roots of a quadratic equation then it can be written as as a product of its linear factors

$$ax^2 + bx + c = a(x - x_1)(x - x_2).$$

If

$$x^2 + px + q = 0,$$

is a quadratic equation with roots x_1, x_2 then

$$x_1 + x_2 = -p, \quad x_1 \cdot x_2 = q.$$

ABSOLUTE VALUE

The absolute value (or modulus) $|a|$ of a real number a is defined such that

1. if $a \geq 0$, then $|a| = a$,
2. if $a < 0$, then $|a| = -a$.

Properties of absolute values

a) $|a| \geq 0$,

b) $|-a| = |a|$,

c) $|ab| = |a| \cdot |b|$,

d) $\sqrt{a^2} = |a|$,

e) $|a| = k \;\Leftrightarrow\; a = k \;\vee\; a = -k$,

f) $|a| < k \;\Leftrightarrow\; -k < a < k \;\Leftrightarrow\; a \in (-k; k)$,

g) $|a| > k \;\Leftrightarrow\; a < -k \;\vee\; a > k \;\Leftrightarrow\; a \in (-\infty; -k) \cup (k; \infty)$.

COMPLEX NUMBERS

A complex number is any number that can be written in the form $z = a + bi$, where $a, b \in \mathbb{R}$ and a is the real part, b is the imaginary part of the complex number and i is the imaginary unit, that satisfies $i^2 = -1$.

Let $z_1 = a + bi$, $z_2 = c + di$ be complex numbers. Then

a) sum $z_1 + z_2 = (a + c) + (b + d)i$,

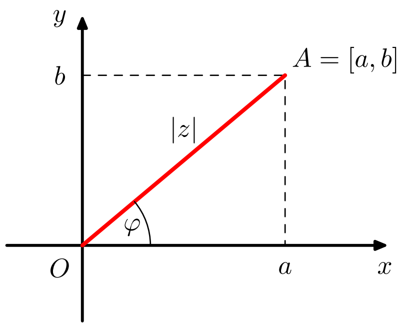
b) difference $z_1 - z_2 = (a - c) + (b - d)i$,

c) product $z_1 \cdot z_2 = (ac - bd) + (ad + bc)i$,

d) ratio $\dfrac{z_1}{z_2} = \dfrac{(a + bi)(c - di)}{(c + di)(c - di)}, \quad c + di \neq 0.$

Note that the complex number $c - di$ is called the complex conjugate of the complex number z_2.

The complex number $z = a + bi$ can be visually represented by the position vector $\overline{OA}$



The absolute value (or modulus or magnitude) is $|z| = \sqrt{a^2 + b^2}$.

$$\cos\varphi = \frac{a}{|z|} \;\Rightarrow\; a = |z| \cdot \cos\varphi$$

$$\sin\varphi = \frac{b}{|z|} \;\Rightarrow\; b = |z| \cdot \sin\varphi$$

Substituing to the algebraic form of a complex number $z = a + bi$ we obtain the polar (trigonometric) form of a complex number

$$z = |z| \cdot (\cos\varphi + i \cdot \sin\varphi).$$

Using Euler's formula

$$e^{i\varphi} = \cos\varphi + i \cdot \sin\varphi$$

we get the exponential form of a complex number

$$z = |z| e^{i\varphi}.$$

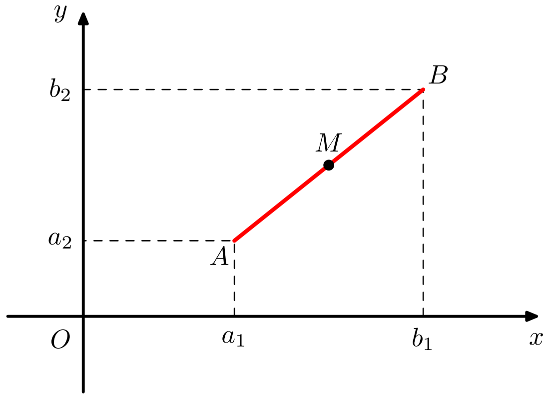
The **distance** between points $A = [a_1, a_2]$ and $B = [b_1, b_2]$ is

$$|AB| = \sqrt{(b_1 - a_1)^2 + (b_2 - a_2)^2}.$$

The vector from point A to point B is $\overrightarrow{AB} = B - A = (b_1 - a_1, b_2 - a_2)$. Then $|AB|$ is also called the length of the vector $\overrightarrow{AB}$.

The **midpoint** of the line segment AB is

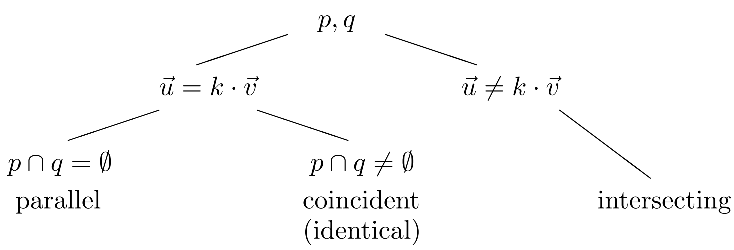
$$M = \left[\frac{a_1 + b_1}{2}, \frac{a_2 + b_2}{2}\right].$$



The equations of a straight line:

1. **parametric form** of the equation –
   the line given by the point $A = [a_1, a_2]$ and the direction vector $\vec{u} = (u_1, u_2)$
   $x = a_1 + t \cdot u_1,$
   $y = a_2 + t \cdot u_2,\ t \in \mathbb{R},$
2. **general form** of the equation –
   $ax + by + c = 0,$
   where $\vec{n} = (a, b)$ is a normal vector of the line, whereby $\vec{n} \perp \vec{u}$,
3. **slope-intercept form** of the equation –
   $y = kx + q,$
   where $k = \tan\alpha$ is the slope while α is the angle between positive part of x-axis and the line and q is the y-intercept.

Relative position of two lines:
Let p be the line given by the point A and the direction vector $\vec{u}$, and let q be the line given by the point B and the direction vector $\vec{v}$. Their relative position can be determined using the following scheme, where $k \in \mathbb{R}$, $k \neq 0$:



The **angle between two** lines:

$$\cos\alpha = \frac{|u_1 v_1 + u_2 v_2|}{|\vec{u}| \cdot |\vec{v}|},$$

where $\vec{u} = (u_1, u_2)$, $\vec{v} = (v_1, v_2)$ and $|\vec{u}| = \sqrt{u_1^2 + u_2^2}, \quad |\vec{v}| = \sqrt{v_1^2 + v_2^2}$.

The **distance of a point** $P = [x_0, y_0]$ from a line $ax + by + c = 0$ is given by:

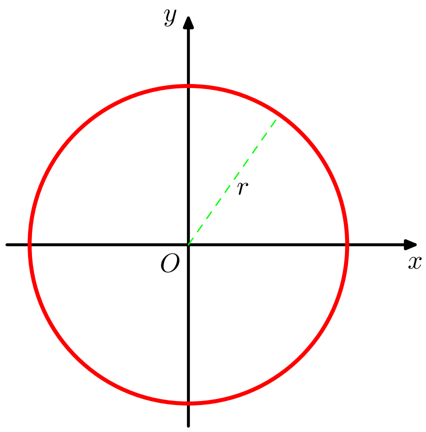
$$d = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}.$$

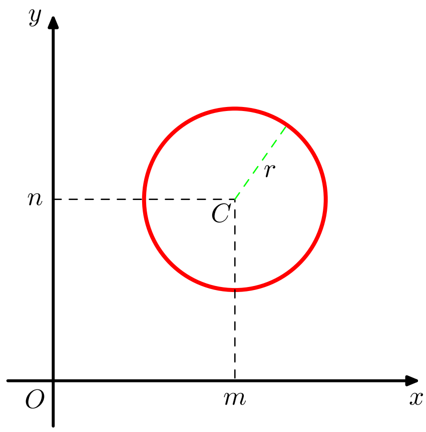
Circle
The standard equation of the circle with center at $C=[0,0]$ and radius $r=|CP|$, where $P=[x,y]$ is an arbitrary point on the circumference of the circle, is

$$x^2+y^2=r^2.$$

If the center is $C=[m,n]$, then the equation is

$$(x-m)^2+(y-n)^2=r^2.$$





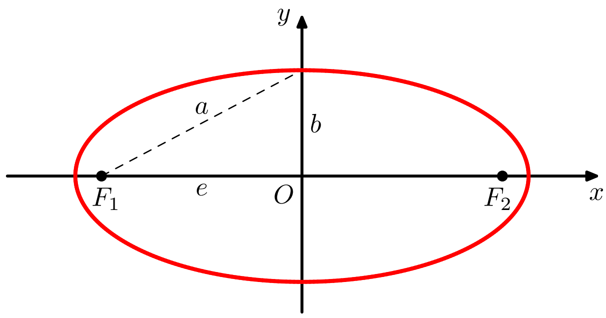
Ellipse
The standard equation of the ellipse, where $P=[x,y]$ is an arbitrary point of the ellipse, is

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, \quad C=[0,0], \quad a^2>b^2,$$

or

$$\frac{(x-m)^2}{a^2}+\frac{(y-n)^2}{b^2}=1, \quad C=[m,n], \quad a^2>b^2.$$

$e=\sqrt{a^2-b^2}$ – eccentricity an ellipse.
$F_1=[-e,0]$, $F_2=[e,0]$ – foci of ellipse.



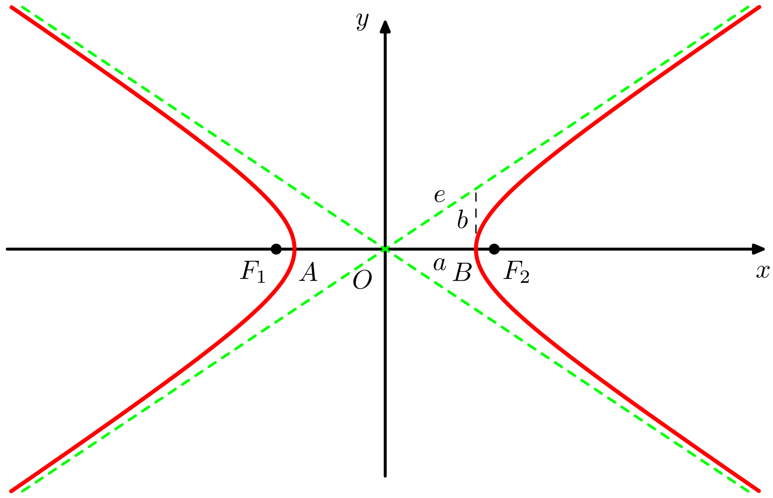
Hyperbola
The standard equation of the hyperbola, where $P=[x,y]$ is an arbitrary point of the hyperbola, is

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1, \quad C=[0,0],$$

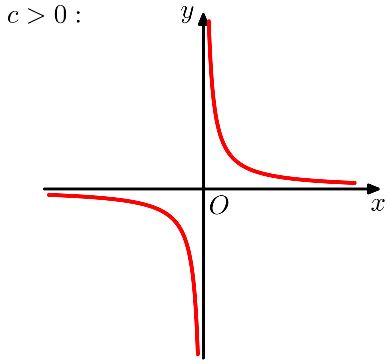
or

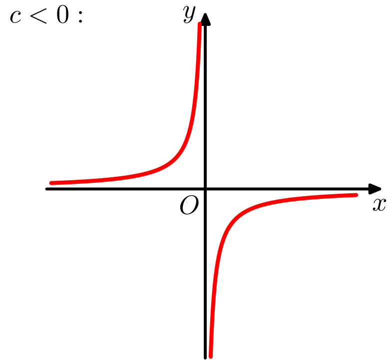
$$\frac{(x-m)^2}{a^2}-\frac{(y-n)^2}{b^2}=1, \quad C=[m,n].$$

$e=\sqrt{a^2+b^2}$ – eccentricity of a hyperbola.
$F_1=[-e,0]$, $F_2=[e,0]$ – foci of hyperbola.
A,B – vertices of hyperbola.
The equations of the asymptotes: $y=\frac{b}{a}\cdot x, \quad y=-\frac{b}{a}\cdot x.$



Rectangular hyperbola $y = \dfrac{c}{x}$

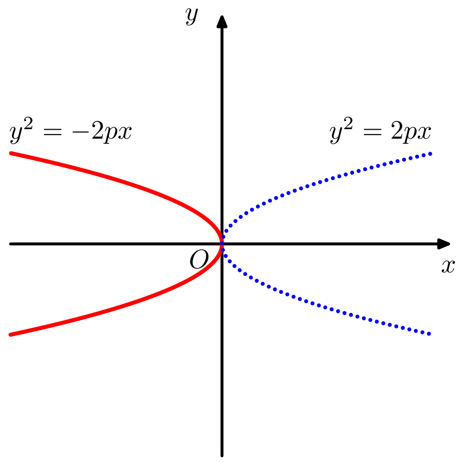


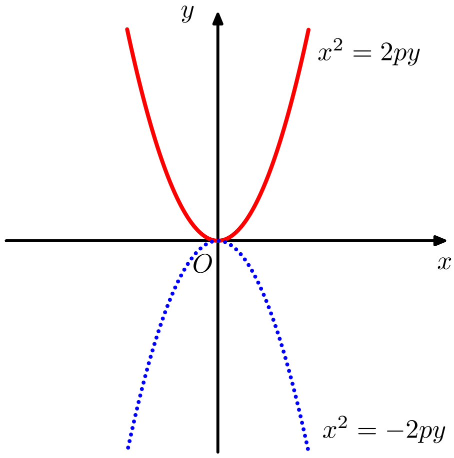


Parabola

The standard equation of the parabola, where $P = [x, y]$ is an arbitrary point of the parabola, is

$$\begin{aligned}
y^2 &= \pm 2px, &\quad &\text{where} \quad p > 0,\ V = [0,0],\ o = x,\\
x^2 &= \pm 2py, &\quad &\text{where} \quad p > 0,\ V = [0,0],\ o = y,\\
(y-n)^2 &= \pm 2p(x-m), &\quad &\text{where} \quad p > 0,\ V = [m,n],\ o \parallel x,\\
(x-m)^2 &= \pm 2p(y-n), &\quad &\text{where} \quad p > 0,\ V = [m,n],\ o \parallel y.
\end{aligned}$$





Differentiation Formulas:

1) $[c]' = 0,$ where c is a constant

2) $[x^\alpha]' = \alpha \cdot x^{\alpha-1}$

3) $[e^x]' = e^x$

4) $[a^x]' = a^x \ln a$

5) $[\ln x]' = \dfrac{1}{x}$

6) $[\log_a x]' = \dfrac{1}{x \ln a}$

7) $[\sin x]' = \cos x$

8) $[\cos x]' = -\sin x$

9) $[\tan x]' = \dfrac{1}{\cos^2 x}$

10) $[\cot x]' = -\dfrac{1}{\sin^2 x}$

11) $[\arcsin x]' = \dfrac{1}{\sqrt{1-x^2}}$

12) $[\arccos x]' = \dfrac{-1}{\sqrt{1-x^2}}$

13) $[\arctan x]' = \dfrac{1}{1+x^2}$

14) $[\text{arccot} x]' = \dfrac{-1}{1+x^2}$

Rules:

$[c \cdot f(x)]' = c \cdot f'(x)$

$[f(x) \pm g(x)]' = f'(x) \pm g'(x)$

$[f(x) \cdot g(x)]' = f'(x) \cdot g(x) + f(x) \cdot g'(x)$

$$\left[\frac{f(x)}{g(x)}\right]' = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{g^2(x)}$$

$[f(g(x))]' = f'(g(x)) \cdot g'(x)$

Integration Formulas:

1) $\displaystyle\int 1 \mathrm{d}x = x + c$

2) $\displaystyle\int x^\alpha \, \mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1} + c$ for $\alpha \neq -1$

3) $\displaystyle\int e^x \, \mathrm{d}x = e^x + c$

4) $\displaystyle\int a^x \, \mathrm{d}x = \frac{a^x}{\ln a} + c$ for $a > 0,\ a \neq 1$

5) $\displaystyle\int \frac{1}{x} \, \mathrm{d}x = \ln|x| + c$

6) $\displaystyle\int \cos x \, \mathrm{d}x = \sin x + c$

7) $\displaystyle\int \sin x \, \mathrm{d}x = -\cos x + c$

8) $\displaystyle\int \frac{1}{\cos^2 x} \, \mathrm{d}x = \tan x + c$

9) $\displaystyle\int \frac{1}{\sin^2 x} \, \mathrm{d}x = -\cot x + c$

10) $\displaystyle\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \begin{cases} \arcsin \frac{x}{a} + c, \\ -\arccos \frac{x}{a} + c \end{cases}$

11) $\displaystyle\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a} \ln \left| \frac{a+x}{a-x} \right| + c$

12) $\displaystyle\int \frac{\mathrm{d}x}{a^2+x^2} = \begin{cases} \frac{1}{a} \arctan \frac{x}{a} + c, \\ -\frac{1}{a} \text{arccot} \frac{x}{a} + c \end{cases}$

13) $\displaystyle\int \frac{\mathrm{d}x}{\sqrt{x^2+k}} = \ln \left| x + \sqrt{x^2+k} \right| + c$

Rules:

$\displaystyle\int c \cdot f(x) \, \mathrm{d}x = c \cdot \int f(x) \, \mathrm{d}x$, where $c \neq 0$

$\displaystyle\int [f(x) \pm g(x)] \, \mathrm{d}x = \int f(x) \, \mathrm{d}x \pm \int g(x) \, \mathrm{d}x$

$\displaystyle\int \frac{f'(x)}{f(x)} \, \mathrm{d}x = \ln|f(x)| + c$

$\displaystyle\int f'(x) \cdot g(x) \, \mathrm{d}x = f(x) \cdot g(x) - \int f(x) \cdot g'(x) \, \mathrm{d}x$